

## PRESSURE IN BULK MATERIAL FLOW – ANGLE OF ENERGY SLOPE

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## TLAK PŘI TOKU SYPKÝCH HMOT – ÚHEL ENERGETICKÉHO SPÁDU

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### Abstrakt

Príspevek popisuje jednu z možností nalezání podobností při toku sypkých granulovaných materiálů. Popis zón, které mají specifické chování pohybu individuálních částic, je utvářen na základě modelu úhlu energetického spádu. Tento model se vztahuje ke kinetické energii částice při počátku a konci sledované trajektorie. Jedná se o model funkce pozice částice v prostoru a jakési kinetické formě pohybu jednotlivých částic.

Tento postup může být s výhodou aplikován za účelem nalezání tlakové špičky v sypké hmotě a jednoduchých závislostí různých granulačních sypkých hmot. Nalezání této tlakové špičky je důležité pro konstrukci zásobníků a skladovacích zařízení.

### Abstract

The article describes a possible procedure for finding similarities in flow of various granular materials. A description of zones possessing a specific mode of motion of individual particulates is performed by means of an angle of energy slope - a relation of kinetic energy of a particulate at the beginning and at the end of an observed particulate trajectory as a function of particulate position in space and a kinetic form of motion of individual particulates.

This procedure can be applied for finding a pressure peak in a flow of bulk material and for finding similarity criteria with various granular materials - for example with particulates of a general shape. Finding a pressure peak is important for constructing silos and storage containers.

**Key words:** flow of bulk materials, kinetic energy, pressure peak, silo and storage containers

### 1. Introduction

Previous phases of scientific knowledge concerning bulk (granular) materials stressed the uniqueness and unrepeatability nature of individual technologies and physical processes going in particulate materials and apparent logical streams of unrelated applications have been arising. The present situation can be characterized as an interdisciplinary and complex approach to solved problems [5]. Sometimes particular applications of a theory in transport, production or storage of bulk materials actually represent studies of changing mechanical, physical and geometrical properties of a material which in some way take into account fundamental physical laws. A particular transport, processing and storage equipment then serves as a boundary condition for finding a solution of a task. The goal is to find flow invariants with respect to various boundary conditions of each real equipment. For example a silo construction represents

a boundary condition for a bulk material discharged through an outlet hole. Such concept supplies a base for the demanded complex and interdisciplinary approach. A definition of the ideal bulk material and particulate flow mechanisms were missing for a realization of this approach. Parameters and operations related to them must be invariant in particular applications and must express (contrary to usual solutions) fundamental laws and properties of bulk materials both in rest and in motion (pressure, flow profiles, dissipated energy).

## 2. Kinetic state of individual particulates approaching to outlet hole (or moving away from outlet hole)

The following paragraphs specify kinematical states which particulates can acquire and a room where a specific movement take place. Possible kinematical states of a single particulate are specified here.

### 2.1 Specification of possible kinetic states of particulate

The way particulates acquire a specific kinetic state of motion is described in detail in [1]. There are presented there the most important features of particulate motion and flow mechanisms by which particulates enter the special modes of motion. Particular particulates both submit themselves to a specific mode of motion of a particular region and create this mode in their movement towards an outlet hole as well.

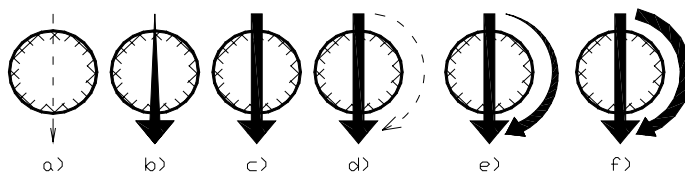


Fig.1 Modes of particulate motion

- a) intermittent translational motion, b) accelerating translational motion, c) uniform translational motion, d) smooth translational and intermittent rotational motion, e) uniform translational and accelerating rotational motion, f) uniform translational and rotational motion

### 2.2 Concept of \*energy slope

The concept of the energy slope is introduced by means of a mathematical formulation reflecting kinetic changes in the process of particulate transition across specific zones.

#### Definition

The tangent of the angle of energy slope  $\alpha$  is defined as a ratio of the kinetic energy of a particulate at the beginning of its motion to the kinetic energy of that particulate at the end of a certain phase of motion. It is a function of a particulate position in relation to the outlet hole.

\*

$$\operatorname{tg}\alpha = \frac{W_{\text{kin1}}}{W_{\text{kin2}}} \quad (1.1)$$

$W_{\text{kin1}}$  - kinetic energy of a particulate at the entrance boundary of a zone the particulate enters

$W_{kin2}$  - kinetic energy of a particulate at the exit boundary of a zone the particulate leaves

### 2.3 Description of zones as characteristic modes of particulate motion

Individual particulates in a flow of granular material move towards an outlet hole. They acquire a characteristic kinetic mode of motion that is defined by their distance to an outlet hole and by properties of bulk material (basic model by [1, 2, 3]).

Not every theoretical possibility of kinetic states must be realized with particulates of a real bulk material. For example materials where the Van der Waals bounds are acting or materials with particulates unable to rotate in their clusters lack the mode of rotation. There are many combinations of bulk material properties nevertheless materials can be characterized by certain generally valid characteristics.

### 3. Phases of particulate movement from trajectory to discharge hole and concept of zone energy slope

A kinetic state of a particulate and practical ways particulates acquire specific modes of motion are described in details in [1] and in Figure 2.

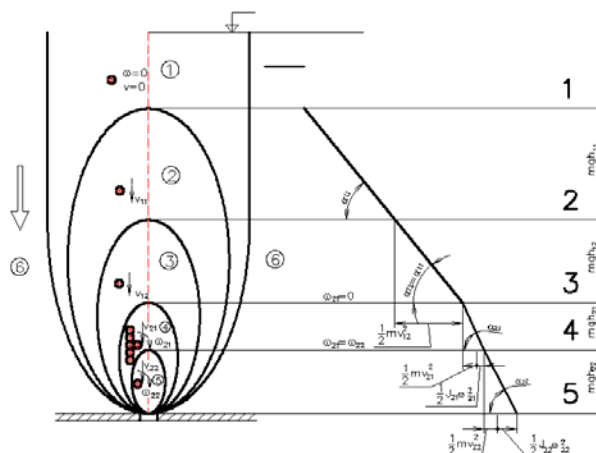


Fig.2 The sequence of specific zones with characteristic movements of particulates and angle of energy slope

In the first phase of flow in the zone no. 1 a particulate exhibits no rotational or translational movement and the energy slope is  $\operatorname{tg}\alpha = 0$ .

In the zone no. 2 a particulate continually acquires translational movement (the particulate has no rotational movement), it is null or non-null at the start of the zone but all particulates exhibit non-null translational movement at the end of the zone. The energy slope is then non-null,  $\operatorname{tg}\alpha = 1$ , or null,  $\operatorname{tg}\alpha = 0$ .

In the zone no. 3 particulates exhibit non-null translational movement and null rotational movement.

The equation of energy conservation (without energy dissipation in ideal case) before and after the movement in this zone is:

$$mgh_{12} = \frac{1}{2}mv_{12}^2 \quad (1.2)$$

The energy slope in the zone no. 3 is:

$$\operatorname{tg}\alpha_1 = \frac{mgh_{12}}{\frac{1}{2}mv_{12}^2} = \frac{\frac{1}{2}mv_{12}^2}{\frac{1}{2}mv_{12}^2} = 1 \quad (1.3)$$

In the zone no. 4 a particulate exhibits non-null translational movement and gradually acquires a rotational movement by its rolling on another particulates. It is bound to a surface of constant energy.

The equation of energy conservation in the zone no. 5 is given by initial energy and final energy in ideal case by the relation:

$$\frac{1}{2}mv_{12}^2 = \frac{1}{2}mv_{21}^2 + \frac{1}{2}J_{21}\omega_{21}^2 \quad (1.4)$$

Angular momentum of inertia of a spherical particulate rolling on a flat surface is:

$$J_{21} = \frac{2}{5}mr^2 + mr^2 = \frac{7}{5}mr^2 \quad (1.5)$$

By using it for a substitution in the previous equation we get

$$\frac{1}{2}mv_{12}^2 = \frac{1}{2}mv_{21}^2 + \frac{1}{2} \cdot \frac{7}{5}mv_{21}^2 \quad (1.6),$$

where  $r^2 \cdot \omega_{21}^2 = v_{21}^2$

This equation yields a relation for entry and exit velocity

$$5v_{12}^2 = 12v_{21}^2$$

The exit velocity of a particulate leaving the zone no. 4 as a function of the entry velocity for the zone no. 4 is:

$$v_{21} = \sqrt{\frac{5}{12}}v_{12} \quad (1.7)$$

The energy slope in the zone no. 4 is

$$\operatorname{tg}\alpha_2 = \frac{\frac{1}{2}mv_{12}^2}{\frac{1}{2}m \frac{5}{12} \cdot v_{12}^2} = \frac{12}{5} = 2,4 \quad (1.8)$$

The energy slope in the zone no. 4 \*is  $\operatorname{tg}\alpha_2 = 2,4$ .

In the zone no. 5 a particulate exhibits both non-null translational and rotational movement. The particulate freely rotates in space.

The equation of energy conservation (before and after the movement) in an ideal case is

$$\frac{1}{2} m \cdot v_{21}^2 = \frac{1}{2} m \cdot v_{22}^2 + \frac{1}{2} J_{22} \omega_{22}^2 \quad (1.9)$$

The angular momentum of inertia of a sphere rotating around its mass center is

$$J_{22} = \frac{2}{5} m \cdot r^2 \quad (1.10)$$

By substituting for the  $J_{22}$  in the previous equation and reordering the parts we get

$$\frac{1}{2} m \cdot v_{21}^2 = \frac{7}{10} m \cdot v_{22}^2 ,$$

$$5v_{21}^2 = 7v_{22}^2 .$$

The exit velocity of the particulate leaving the zone no. 4 serves as an entry velocity into the zone no. 5 and

$$v_{22} = \sqrt{\frac{5}{7}} v_{21} \quad (1.11)$$

The energy slope in the zone no. 5 is expressed as

$$\operatorname{tg} \alpha_3 = \frac{\frac{1}{2} m \cdot v_{21}^2}{\frac{1}{2} \cdot m \cdot \frac{5}{7} \cdot v_{21}^2} = \frac{7}{5} = 1,4 . \quad (1.12)$$

#### 4. Application

The described laws are made apparent by an increase of pressure in the zone of starting particulate rotation. This is caused by a higher consumption of potential energy in forcing particulates into rotation (the particulates already possess a translational mode of motion). The result of this process is an increase of space filling in the zone no. 4 and loosening of material in the zone no. 5 due to the change of flow mechanism [1, 2, 3]. In accordance with the previous chapter in case of spherical particulates the pressure between the  $h_{21}$  and  $h_{22}$  heights above an outlet hole is equal to the 2.4 multiple of the static pressure by Janssen (zone no. 4). The increase of pressure produces an increase of material density (the increased degree of \*compactification of material) in the specified room. The increase of pressure in the zone no. 4 was proved many times but its interpretation was different. That is why a new verifying

experiment was focused on the measuring of an increase of material density (the coefficient of space filling) in the zone no. 4.

#### *4.1 Densification of bulk material above discharge hole*

In accordance with the aim of the project the second of the authors realized a measurement to verify the expectations of the theory devised at the TU Ostrava 1988 [1] and since then verified in specific technological applications in the laboratory of \*Prof. Dr.-Ing. J.Schwedes, Institut für Mechanische Verfahrenstechnik der TU Braunschweig.

The work describing an origin and characteristic features of two basic mechanisms of bulk material flow published in [1, 2, 3, 4] implies that energy consumption in the specific flow mechanisms is different. The energy consumption of the shell flow mechanism in the zone no. 4 is 2.4 multiple of the basic energy consumption, as deduced for the no. 4 flow zone in the previous chapter. The energy slope  $\text{tg}\alpha_2 = 2,4$ . Because of the fact that no other kind of energy with an exception of pressure energy can increase, this dynamic change when compared with the static situation, can be expected. The result of this shell flow mechanism that leads to the rotation of particulates [1] is the expected considerable increase of poured material density. This densification is caused by ordering of particulates in the zone and proportional to a pressure increase.

### **Experiment description**

The method of the Particle Image Velocimetry (PIV) was used in the experiment. The method consists in dividing the scanned field into imaginary squares and software tries to localize a defined square in a succession of pictures and computes a flow velocity by means of two positions and a time interval between them. An inner algorithm of the software computes a probability of the fact that some positions of squares on a successive picture represent a new position of the original square. The R ratios of various probabilities give a new position of the original square,  $Q = \text{Probability}_1 / \text{Probability}_2$ .

In case that the R ratio is much higher than 1, the position 1 is a new position of the original square. In case the Q ratio is close to 1, the new position is not unambiguously discernible. It could happen because of faulty conditions in shooting sequence - light reflections on silo walls, a bad choice of bulk material etc. (It is known for example that polyethylene (PET) grains always produced better results than polypropylene (PP) grains because the former were not transparent.

The computations produce a vector field of velocities over the silo. A shortage of this method is the fact that it supplies only the velocity field on the silo walls.

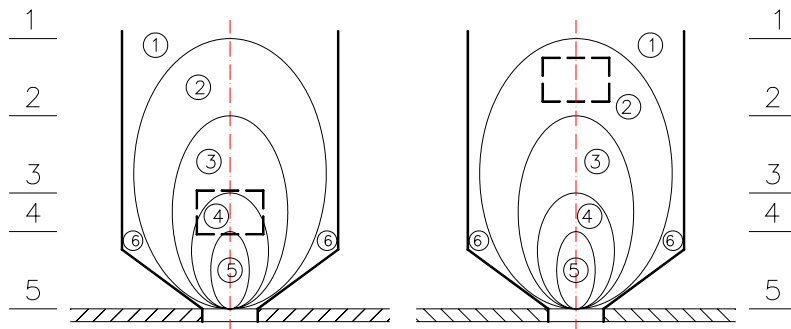
The classic formula for the evaluation of a bulk material density in the room of an outlet hole was used, that means  $Q = S \cdot v \cdot \rho$  [kg.sec<sup>-1</sup>].

An assumption was accepted that the velocity found by PIV method is the same as the real outlet velocity in the outlet hole (wall-surface versus volume velocities).

The out flowing bulk material was caught into a container within 5 second intervals and then weighted. Because parameters had no changed, the density of bulk material in the place of the outlet hole could be evaluated. The outlet hole of two dimensions was used - 25 x 25 mm and 40 x 40 mm and a series of 5 measurements for the both two outlet holes were performed. An arithmetic average of densities was computed then.

The overall density of PET particulates in the zone no. 4 inside silo was  $\rho_1 = 844.24$  kg.m<sup>-3</sup> and in the zone no. 5 above the outlet hole this value was  $\rho_2 = 457$  kg.m<sup>-3</sup> in average (the

results differ with the size of tested outlet holes). The measurement means that in the zone no. 4 above the outlet hole the mass of particulates in the unit volume was 1.847 times higher than in



the place directly above the outlet hole. The ratio of the densities is  $\rho_1/\rho_2 = 1.847$ . We explain it here by a change of the flow mechanism.

Fig.4 Scanned zones

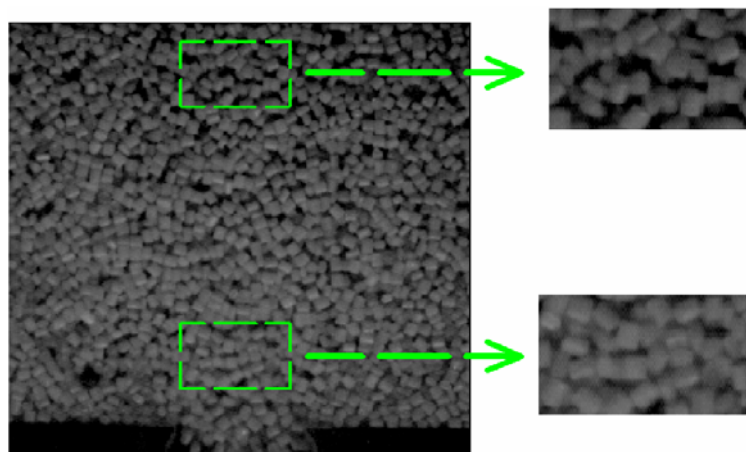


Fig.5 Photography of measured regions

## 5. Conclusions

In approaching to an outlet hole the bulk material crosses zones where changes of kinetic states of particulates take place. The kinetic state of a particulate is bound with a flow mechanism. There are two basic flow mechanisms differing in translational and rotational movement of particulates. The changes of kinetic states of particulates take place at boundaries of flow fields. Sets of particulates having the same mechanism of flow form a flow field and its particulates keep the same kinetic state. A specific flow field with a typical particulate movement is always preceded by a transitional zone where particulates acquire proper specific mode of motion. That means that two specific zones and two transitional zones exist above an outlet hole.

There are four important regions related to flow mechanism and thus forming four important flow profiles. The particulates in surrounding room which is not affected by these four

flow profiles (zone no. 1) remain relatively in rest and sometimes this material is called the sagging of bulk material. The bulk material far-off of flow profiles 2 - 5, and neither above flow profiles in the zone no. 1, are out of reach of the outlet hole. In practice this regions are called the dead zone. Thus we can see that there are altogether six different zones with specific modes of motion.

In the upper zone no. 1 particulates are sagging into a room left by particulates below them. In the zone no. 2 particulates acquire a translational movement and in the zone no.3 the translational movement becomes steady. In the zone no. 4 particulates acquire a rotational mode of motion (by rolling on other layers) and in the zone no. 5 the full kinetics of motion is developed - the translation and rotation. In the dead zone no. 6 the particulates are in rest but can slide into flow profiles 1-5.

The specific zones can be visually described in case of the gravitational flow where an outlet hole is positioned directly under a column of bulk material and particulates are "flowing" directly to an outlet hole.

Not all flow zones must always manifest themselves in technical practice. For example in case of a bulk material consisting of particulates that are not able to rotate in great aggregations no zone no. 4 and no. 5 appear.

A pressure peak with particulates of other forms than spheres can be found by a procedure similar to one presented here. That means by computing an angular momentum of inertia and then an angle of the energy slope. Inhomogeneous sets of particulates need a specification of an equivalent particulate and the application of standard procedure.

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### Nomenclature

$\alpha$	angle of energy drop, index belongs to a zone	[ deg ]
$W_{kin1}$	starting particulate kinetic energy in a zone	[ J ]
$W_{kin2}$	final particulate kinetic energy in a zone	[ J ]
$v$	velocity of straight line movement of a particulate	[ m·s <sup>-1</sup> ]
$\omega_{21}$	angle velocity of a particulate in zone of generating general movement	[ rad·s <sup>-1</sup> ]
$\omega_{22}$	angle velocity of a particulate in a zone of developed general movement	[ rad·s <sup>-1</sup> ]
$g$	gravity acceleration	[ m·s <sup>-2</sup> ]
$m$	mass of a particulate	[ kg ]
$h$	height of a specific zone of movement	[ m ]
$r$	radius of a particulate	[ m ]
$J_{12}$	angular momentum of inertia of a particulate in zone of generating rotational movement	[ kg·m <sup>2</sup> ]
$J_{22}$	angular omentum of inertia of a particulate in zone of developed general movement (rotation and translation)	[ kg·m <sup>2</sup> ]
$v_{11}$	particulate velocity in zone of starting straightline movement	[ m·s <sup>-1</sup> ]
$v_{12}$	particulate velocity in zone of developed straightline movement	[ m·s <sup>-1</sup> ]
$v_{21}$	particulate velocity in zone of starting general movement	[ m·s <sup>-1</sup> ]
$v_{22}$	particulate velocity in zone of developed general movement	[ m·s <sup>-1</sup> ]



S	size of outlet hole aperture	[ m <sup>2</sup> ]
$\rho$	density	[ kg·m <sup>-3</sup> ]

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