

ANGULAR VIBRATION MEASUREMENTS OF THE POWER DRIVING SYSTEMS

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UHLOVÉ VYBRAČNÉ MIEŠANIE MOTORICKÝCH POHONOVÝCH SYSTÉMOV

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UHLOVÉ VYBRAČNÉ MERANIE MOTORICKÝCH POHONOVÝCH SYSTÉMOV

Abstrakt

Článok sa zaoberá problémom meraní veľkosti uhlovej vibrácie v zmysle uhlu otáčania, uhlovej rýchlosti alebo uhloveho zrýchlenia. Informačný zdroj o uhlových vibráciach je prírastkový rotačný kódovač produkujúci vlákno z impulzov alebo ďalší impulzný generátor. To je predpoklad, že frekvencia impulzov je zmenená rovnomerne k frekvencii otáčania. Čas záznamu od impulzného signálu sa transformuje do analytického signálu použitím Hilbertovej transformácie. Odvíjacia fáza z analytického signálu dáva uhol otáčania, ktorý je zaznamenaný k hodnoteniu uhlovej rýchlosti a uhloveho zrýchlenia. Signály, ktoré sú rušené hlučnosťou sú zosilnené filtrovaním vo frekvenčnej doméne. Spôsob merania opísaný dole, môže byť použitý pre sledovanie pohonných systémov valcovacích stolíc pre tenké plechy v hutníckom priemysle. Uhlové vibračné meracie techniky môžu prispieť k zlepšeniu akosti povrchu plechov. Práca tohoto nového spracovania údajov je demonštrovaná na chybe ozubeného prevodu a uhlovom meraní vibrácií kľukového hriadeľa automobilového motora.

Abstract

The paper deals with the problem of angular vibration measurements in terms of rotation angle, angular velocity or angular acceleration. The source of information about angular vibrations is an incremental rotary encoder producing a string of impulses or another impulse generator. It is supposed that the impulse frequency is changed proportionally to the rotational frequency. A time record of an impulse signal is transformed to an analytical signal using Hilbert transform. The unwrapped phase of the analytical signal gives the rotation angle, which is employed to evaluation of angular velocity and angular acceleration. Signals that are disturbed by noise are enhanced by filtration in the frequency domain. The measurement method described below can be used for inspection of the driving systems for sheet rolling mills in metallurgy industry. Angular vibration measurements can contribute to improve the sheet surface quality. Employing of this new data processing tool is demonstrated on gear transmission error and car engine crankshaft angular vibration measurements.

Key words: Angular vibration, phase demodulation, Hilbert transform, phase unwrapping, impulse signal, transmission error

Introduction

Rotational speed is measured in terms of the number of revolutions per minute (RPM) while the torsional vibration is measured in terms of the angle, angular velocity or angular acceleration. The uniform rotational speed at the constant value of RPM corresponds to growing up the shaft angle proportionally to the elapsed time. The angle time history, having the form of the sum of a term that is depending linearly on time and a term that is randomly or regularly varying in time around zero, results from angular vibration during rotation. The angular velocity is obtained as the first derivative of the angle while the angular acceleration is evaluated as the second derivative of the angle. The torsional vibration can be for instant measured by tangentially mounted accelerometers laser torsional vibration meter based on Doppler effect incremental rotary encoders producing several hundreds of pulses per revolution

Shaft encoders give usually a train of pulses, rather than a sinusoid. The simplest method for evaluation of the instantaneous rotational speed is the reciprocal value of the time between two consecutive pulses. Time interval length measurement is performed either by counting a string of pulses produced by a high frequency oscillator (100 MHz) and triggered by encoder signal or by counting the sample number between adjacent pulses. In case of the sample counting the time interval is improved by interpolation, which results in some 50 times more accurately measurement than indicated by the sample number. The accuracy in this case is satisfying for the RPM measurement based on only one pulse per shaft rotation.

In the case of the large number of pulses per revolution, another method based on the phase demodulation is employed [2]. The pulse signal consists of several harmonics of the basic pulse frequency. Each of the harmonic components is the carrier component that can be modulated by varying rotational speed. An example of the phase-modulated signal is shown in figure 1.

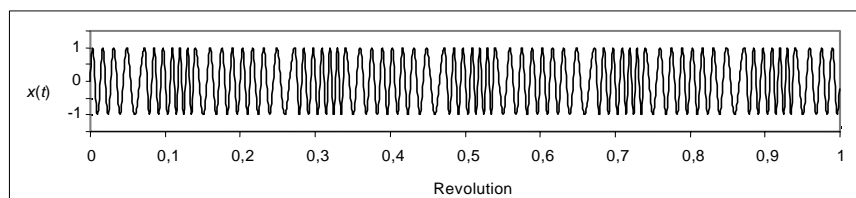


Fig.1 Phase-modulated signal

The phase modulation signal can be derived from the phase of an analytical signal that is evaluated using the Hilbert Transform technique. To compound the complex analytical signal $z(t)$ the real sampled signal $x(t)$ must be extended by an imaginary part $y(t)$ that is the mentioned Hilbert Transform of the real signal.

$$z(t) = x(t) + jy(t) = |z(t)| \exp(j\phi(t)), \quad (1)$$

To transform the signal $x(t)$ to $y(t)$ the following two methods can be employed FFT (Fast Fourier Transform) Real time digital filters

The method based on FFT uses the relationship between the components X_i and Y_i , which are corresponding to the sampled signal x_i and y_i , where $i = 0, 1, \dots, N-1$, benefiting from the following formula

$$Y_i = j \operatorname{sign}(N/2 - i) X_i \quad (2)$$

The approach based on the digital filter is described in figure 2. The digital filter, called Hilbert transformer, performs the phase shift by the angle of $\pi/2$.

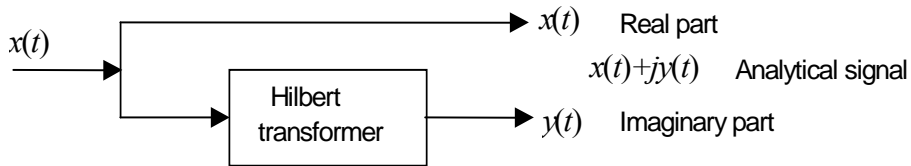


Fig.2 Evaluation of the analytical complex signal in real time

As the angle $\varphi_i = \arctg(y_i/x_i)$ of the complex values ranges only from $-\pi$ to $+\pi$ the time function contains jumps at $-\pi$ or $+\pi$ (see figure 3). The true angle of the analytical signal must be extracted by unwrapping that is based on the fact that the absolute value of the difference between two consecutive angle samples is less than π

$$\Delta\varphi_i = \varphi_i - \varphi_{i-1} < -\pi \Rightarrow \varphi_i + 2\pi \rightarrow \varphi_i, \quad \Delta\varphi_i > +\pi \Rightarrow \varphi_i - 2\pi \rightarrow \varphi_i. \quad (3)$$

This property results from the Shannon sampling theorem.

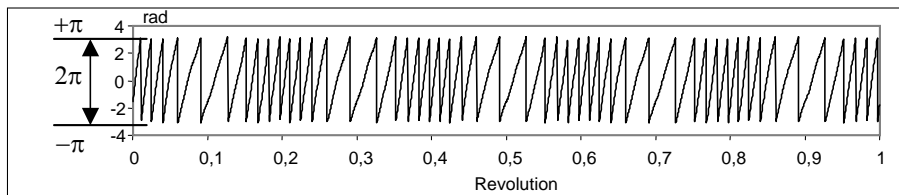


Fig.3 Phase of analytical signal ranging in interval from $-\pi$ to $+\pi$

The principle of phase unwrapping for a harmonic signal on the diagram in figure 1, which is modulated by another harmonic signal, is shown in figure 4. The relationship between the phase of the analytical signal and the phase modulation signal $\varphi_M(t)$ is as follows

$$\varphi(t) = \omega_0 t + \varphi_M(t), \quad (4)$$

where ω_0 is an angular frequency of the carrier component. The first derivative of the linear $\omega_0 t$ term with respect to time t corresponds to the steady-state rotational speed.

Transmission Error Measurements

Noise and vibration problems in gearing are mainly concerned with the smoothness of the drive. The parameter that is employed to measure smoothness is a Transmission Error (T. E.). This parameter can be expressed as a linear displacement at a base circle radius defined by the difference of the output gear's position from where it would be if the gear teeth were perfect and infinitely stiff. Many references have attested to the fact that a major goal in reducing gear noise is to reduce the transmission error of a gear set.

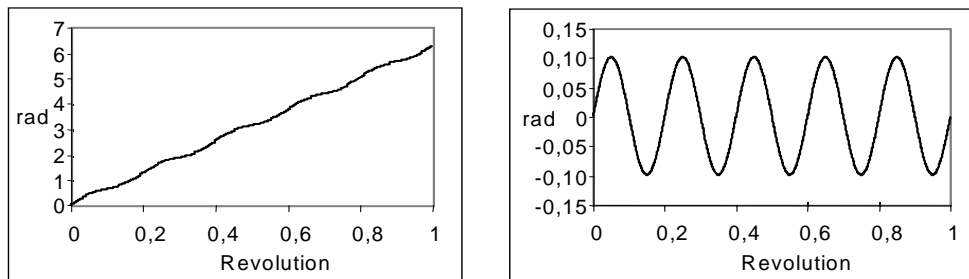


Fig.4 Unwrapped and removed linear trend in phase of analytical signal

The basic equation for T.E. of a simple gear set is given as

$$TE(m) = \left(\Theta_2 - \frac{n_2}{n_1} \Theta_1 \right) r_2 \quad (5)$$

where n_1 , n_2 are teeth numbers of pinion and wheel respectively, Θ_1 , Θ_2 are angles of rotation of the mentioned gears and r_2 is a wheel radius.

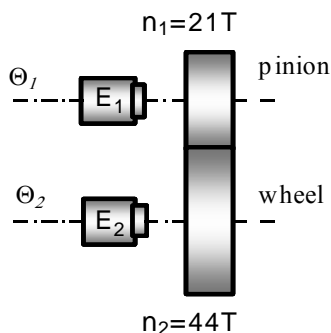


Fig.5 Measurement arrangement

T.E. results not only from manufacturing inaccuracies such as profile errors, tooth pitch errors and run-out, but from a bad design. The pure tooth involute deflects under load due to the finite mesh stiffness caused by tooth deflection. A gearbox and shaft system deflects due to load as well. While running under load one of very important parameters, tooth contact stiffness, is varying what excites the parametric vibration and consequently noise.

The sketch of the gear set consisting of the 21- and 44-tooth gears under test and attached incremental rotary

encoders, designated by E1 and E2 is shown in figure 5.

Both the encoders are of Heidenhain origin, the ERN 460-500 type. A perfectly uniform rotation of gear produces an encoder signal having in its frequency spectrum a single component at the frequency that is a multiple of the gear rotational frequency. As both the encoders generate 500 impulses per encoder rotation, the frequency of the single components in orders (a multiple of the encoder rotational frequency) is equal to the same number as the number of the impulses.

Pulse signals from encoders are recorded by PULSE, the Brüel & Kjær signal analyzer. To simplify the phase demodulation an Order Analyzer instrument was employed which resulted in time history records corresponding to one complete gear revolution. A method of synchronized averaging in the time domain was employed for reducing random noise in the measured data. As it is known, the order analysis is based on data resampling in such a way, that sampling frequency follows the mean frequency of shaft rotational speed during one complete shaft revolution. The mean rotational frequency is evaluated by means a train of pulses generated once per a shaft revolution. Therefore the pulses distribution inside this time record gives information about the instantaneous rotation angle of each of the gears under test. As a

consequence of Shannon's sampling theorem a few pulses must be recorded during each mesh cycle. It means that the number of pulses produced per encoder revolution must be a multiple of the tooth number. If five harmonics of toothmeshing frequency are required then the number of pulses per gear revolution must be at least ten times higher than the number of teeth. The encoder generating 500 pulses per revolution seems to be an optimum. The length of resampled time record equals to 2048 samples per gear revolution. The sample number is a power of two, which is required by FFT and in corresponding order spectrum, ranging to 800 orders, there is a space for ± 300 sideband components around the carrying component of 500 orders in a frequency spectrum. The phase modulation gives rise to sidebands around the carrying frequency in the frequency spectrum of the modulated harmonic signal. The frequency range of the mentioned Order Analyzer in the described conditions limits the gear rotational speed to the value of 1900 RPM at the sampling frequency of 65536 Hz. Gear loading has not any influence on the discussed sampling problem.

Results of T.E. measurements

The gear speed variation results in the phase modulation of the impulse signal base frequency. As noted above the phase-modulated signal contains sideband components around the carrying component. The distance of the dominating sideband components from the carrying components equals to the integer multiple of the tooth number as it is shown in figures 6. The frequency scale of both the frequency spectra is in order; it means the multiples of the gear rotational frequency. The frequency of the carrying component is equal to 500 orders while the sideband component associated with the corresponding gear is at the distance of $\pm 21k$ or $\pm 44k$ (where $k=1, 2, \dots$) order units from the mentioned carrying component frequency. Take notice of the fact that the dominating components in both the sidebands exceed the background noise level at least 10 times or even more. Both the spectra were evaluated from time signals that are a result of synchronized averaging of 100 revolutions of gears under test.

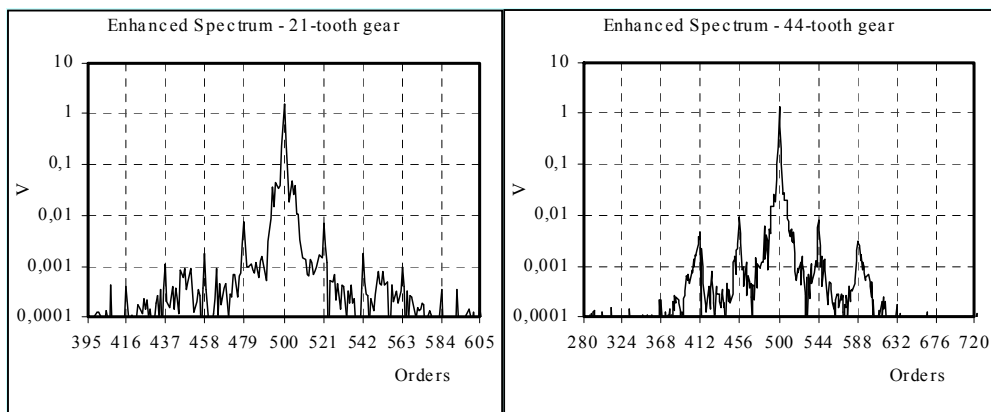


Fig.6 Frequency spectrum of phase modulated signal generated by the E1 and E2 encoders

The phase modulation signal in degrees during the pinion revolution is shown in figure 7. The enhanced signal contains five harmonics of the toothmeshing frequency, each of

them with 3 pairs of sidebands for the 21-tooth gear and with 6 pairs of sidebands for the 44-tooth gear respectively. The sideband components contain information about the phase modulation of angle variation. When all these sidebands are removed a purely periodic signal is obtained. The filtration in the frequency domain can be considered as an averaging of the second stage. Therefore, one of these periods corresponding to the gear tooth pitch rotation can be taken as a representative to characterize angular vibration in average. The result of mentioned averaging is called the average toothsmesh. The term “averaged toothsmesh” was introduced to associate vibration and noise measurement with a gear design [2].

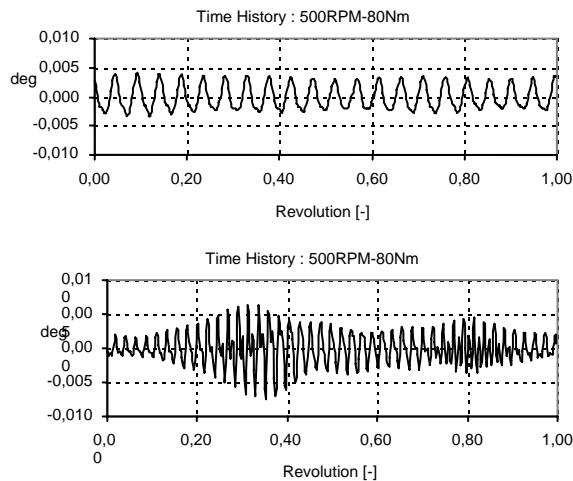


Fig.7 The second stage of angular vibration averaging for the 21-tooth gear

The same average toothsmesh in angular variation can be evaluated for the 44-tooth gear. Angle variation can be easily transformed into the arc length variation. The difference between both the angle variations gives the transmission error. The only problem consists in the true phase delay between these periodic signals because the signals from the encoders are recorded separately. Solving of this problem is based on the similarity of responses both the gears to dynamic forces acting between mating teeth, for instance in acceleration of some point on the gearcase. Both the encoder pulse signals are sampled together with the acceleration signal. Two-stage averaging of the twice-measured acceleration signal gives average toothsmesh responses that are delayed. The lag for the maximum correlation gives the relative delay.

T.E. is given as the difference between the angular vibration signals, it means the length of the pinion arc minus wheel arc produced by the mating gears. The result is shown in figure 8. All the experimental data was taken from a car

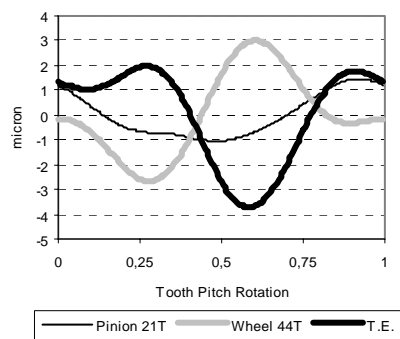


Fig.8 Transmission error against rotation angle in range of the tooth pitch

gearbox. The results correspond to the rotational speed of 500 RPM at the input shaft and almost full load. The measurement method was tested at the maximum rotational speed of 1250 RPM.

If the output signals, which are produced by the mentioned two encoders attached to the gear pair, are recorded simultaneously then the real time method for evaluation of the angle difference can be employed for T.E. estimation. As it was described above the phase demodulation of the pulse string signals result in the unwrapped angle as a function of time. When the revolution angle of the driven gear is corrected by taking into account the gear ratio then both the time functions can be subtracted to evaluate the angle difference and finally transmission error using the pitch circle diameter. The example of the analysis result for the gear set consisting of the 15-tooth pinion and the 68-tooth wheel is shown in figure 9. The pinion was rotating at 400 RPM.

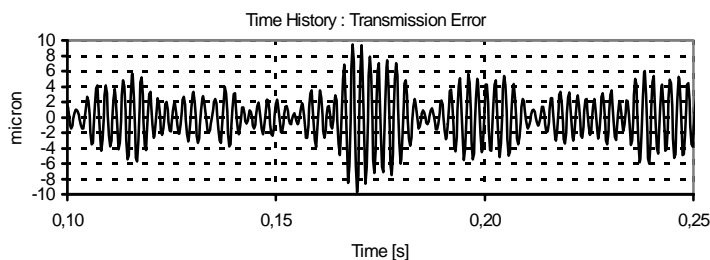


Fig.9 Angle rotation difference as a function of time

Engine crankshaft angular vibration measurements

Non-stationary car shaking, which can be designated as well as a burst vibration repeating randomly, is observable for instance when a car is waiting before traffic lights and engine is running at idle. Passengers notice this phenomenon and are disturbed or suspicious that something is wrong or out of order. So it is important to control this phenomenon at newly produced cars and do not allowed to exceed this kind of vibration out of an acceptable limit. It must be said as well that the vibration level is negligible from the point of human exposure to the whole-body vibration.

Rotational speed of the 4-stroke / 4-cylinder spark engines running at idle varies in a certain range at the average level of 800. The purpose of measurements is to explain the source of the rotational speed non-uniformity. The first step to analysis is to identify the rotational speed variation not only in term of the complete revolutions but in terms of the basic operational stages of the engine under test. This goal of tests requires the measurement of the instantaneous rotational speed and angular acceleration.

Measurements were restricted only to the time history of a pulse train that is generated by a transducer that is connected to the engine control unit. Any special device or encoder is not supposed to attach to the engine crankshaft. The transducer that is a part of engine generates 58 pulses between the gaps of 2 missing pulses. All the 58 pulses are distributed in the period of a revolution uniformly in 60 positions situated proportionally to the rotational angle. As the operational cycle consists of two revolutions the time history of a pulse signal is shown in figure 10. To improve accuracy of the modulation signal evaluation a computer program incorporates the missing pulses.

Angular velocity and acceleration were evaluated using the first and second derivative of the crankshaft angle with respect to time, respectively. Differentiation was performed in the frequency domain in such way that the FFT angle spectrum was multiplied by the term of $j\omega$ or

$(j\omega)^2$. As multiplication by mentioned terms amplifies the high frequency noise in the measurement data proportionally to the frequency or even proportionally to the square of the frequency the filtration in the time domain was employed. The spectrum components with the frequency higher than the 6th order of the rotational frequency were put to the zero. The inverse FFT results in the time history of angular velocity or acceleration (see figure 11).

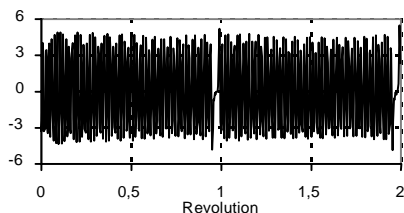


Fig.10 Time history of impulse signal

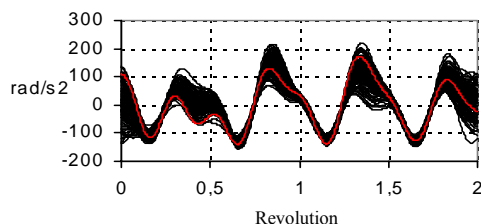


Fig.11 Time history of angular acceleration

Conclusion

The paper reviews the field of diagnostics based on the measurement of angular vibration, which gives useful information about the operational condition of machines. The first example is focused on the problem of the gearbox transmission error measurement. The instantaneous value of error results from variation of the gear angle revolution from a linear term depending on steady state rotation while the second example presents results of the car engine crankshaft instantaneous rotational speed and acceleration measurements. The source of information about angular vibrations is a string of impulses with the frequency proportional to the rotational speed. The measurement method is based on the phase demodulation of impulse signals using the theory of the analytical signals.

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