THE INFLUENCE OF USING THE DIFFERENT NUMERICAL METHODS TO FORECAST THE ALUMINIUM PRICES BY MEANS OF TWO INITIAL VALUES

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VPLYV POUŽITIA RÔZNYCH NUMERICKÝCH METÓD PRI PROGNÓZOVANÍ CIEN HLINÍKA VYUŽITÍM DVOCH ZNÁMYCH VSTUPNÝCH HODNÔT

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Abstrakt

Hodnotenie a predpovedanie pohybu cien na komoditných burzách predstavuje aj v súčasnosti veľmi aktuálnu problematiku. Existuje množstvo prístupov k predpovedaniu pohybu cien, medzi inými aj prístupy založené na matematických modeloch. Prognózovanie je často založené na štatistických modeloch, ktoré však vyžadujú množstvo historických údajov [1], [5]. Množstvo požadovaných údajov môže byť v niektorých prípadoch problémom. Za týchto okolností je vhodné využiť matematické metódy požadujúce menší počet historických údajov.

Článok predkladá odvodenie numerického prognostického modelu založeného na numerickom riešení Cauchyho začiatočnej úlohy pre obyčajné diferenciálne rovnice 1. rádu. Účelom modelu je prognózovanie cien hliníka na Londýnskej burze kovov. Zaoberali sme sa mesačnými priemermi denných uzatváracích cien hliníka „Cash Seller&Settlement price“ v období december 2002 až jún 2006.

Prognózy cien hliníka určujeme ako numerické riešenia zvoleného typu Cauchyho začiatočnej úlohy pomocou dvoch vybraných numerických metód. Počas prognózovania mesačných priemerov cien hliníka porovnávame presnosť prognóz získaných týmito vybranými numerickými metódami. Metóda exponenciálneho prognózovania je teoreticky odvodená v [4]. Za účelom porovnania presnosti prognózovania touto metódou používame súbežne aj všeobecne uznávanú numerickú metódu vložených formúl Rungeho-Kutta 5. rádu do 6. rádu [2], [3]. V závislosti od spôsobu výpočtu prognózy využitím dvoch známych predchádzajúcich hodnôt uvažujeme dva typy prognózovania, denné prognózovanie a mesačné prognózovanie. V článku je analyzovaná aj úspešnosť numerických metód v závislosti od priebehu vývoja ceny hliníka.

Porovnaním hodnôt prognóz určených numerickými metódami a burzových cien v sledovaných mesiacoch sme zistili, že rozdiely v prognózovaní využitím rozdielnych zvolených numerických metód buď neexistujú (denné prognózovanie) alebo sú minimálne (mesačné prognózovanie). Mesačným prognózovaním metódou exponenciálneho prognózovania získame zväčša väčšie hodnoty prognóz ako mesačným prognózovaním vloženými formulami Rungeho-Kutta 5. rádu do 6. rádu. Na základe toho, prognózovanie metódou exponenciálneho prognózovania je väčšinou výhodnejšie v rastovom trende, ak rast zvyšuje svoju prudkosť, v miernejšom klesajúcom a v klesajúcom trende s rastom prognózovanej ceny. Naopak, vložené formule Rungeho-Kutta 5. rádu do 6. rádu poskytujú zväčša presnejšie prognózy v prudšom klesajúcom trende, v miernejšom rastúcom trende a v rastovom trende, keď prognózovaná cena klesá.

Abstract

Observing trends and forecasting movements of the metal prices is still a current problem. In the mathematical models forecasting the prices on the commodity exchanges the statistical methods are usually used [1], [5]. They need to process a large number of the historical market data. The amount of the needed market data can sometimes be a problem. In such cases other mathematical methods are required.

The paper deals with deriving the numerical model based on the numerical solution of the Cauchy initial problem for the 1st order ordinary differential equations to prognose the prices of aluminium on the London Metal Exchange. We came out of the monthly averages of the daily closing aluminium prices "Cash Seller&Settlement price" in the period from December 2002 to June 2006.

When forecasting monthly average prices, we compare the accuracy of the prognoses acquired by the two chosen numerical methods. In the paper the method of exponential forecasting and the embedded Runge-Kutta formulae of the 5th order to the 6th order are used. The method of exponential forecasting was derived in [4]. In this paper its forecasting success is observed in comparison with the well-known numerical method (the embedded Runge-Kutta formulae of the 5th order to the 6th order [2], [3]). Two types of forecasting are created according to the way of calculating the prognosis by means of the two known previous values, daily forecasting and monthly forecasting. The advantages of the numerical methods during different movements of the aluminium prices are analysed.

Comparing the prognoses obtained by means of the chosen numerical methods and the aluminium stock exchanges we have found out that there are either no diferences in forecasting by using the chosen numerical methods (daily forecasting) or, if there are any, they are little (monthly forecasting). By monthly forecasting, using the method of exponential forecasting we obtain usually higher values of the prognoses than by using the embedded Runge-Kutta formulae of the 5th order to the 6th order. Based on this knowledge, forecasting by using the method of exponential forecasting is usually more advantageous in the stable increasing trend if the increase of the price raises its intensity, during the slighter decline of the prices and in the period of the change from the decreasing trend to the increasing one. On the other hand, the embedded Runge-Kutta formulae of the 5th order to the 6th order gives usually more accurate prognoses in the rapid decreasing trend, in the case of the decreasing intensity of the increase in the prices and in the increasing trend with the decline of the prognosed price.

Keywords: forecasting, numerical modelling, ordinary differential equation

1. Introduction

One of the most important factors determining the prices of the non-ferrous metals is the impact of the London Metal Exchange (LME). It is the world's premier non-ferrous metals market. The origins of LME can be traced back as far as the opening of the Royal Exchange in London in 1571. Although there are other commodity exchanges where metals are traded (for example NYMEX in the USA or SIMEX in Singapure) the majority of the producers and the bussinessmen respect just the official prices daily closed on LME.

Observing trends and forecasting movements of the metal prices is still a current problem. There are a lot of approaches to forecasting the price movements. Some of them are based on mathematical models. Forecasting the prices on the commodity exchanges often uses the statistical methods that need to process a large number of the historical market data. The amount of the needed market data can sometimes be a disadvantage. In such cases other mathematical methods are required.

We have decided to use the numerical methods. Their advantage is that much less market data is needed in comparison with the statistical models. Our numerical model for forecasting prices is based on the numerical solution of the Cauchy initial problem for the 1st order ordinary differential equations.

Let us consider the Cauchy initial problem in the form

$$y' = f(x, y), \; y(x_0) = y_0 . \tag{1}$$

We assume that there exists just one solution $y(x)$ of the problem (1) in the interval $\langle \alpha, \beta \rangle$, which has an exponential character. Based on this assumption we shall consider the Cauchy initial problem in the form

$$y' = a_1 y, \; y(x_0) = y_0 . \tag{2}$$

The particular solution of the problem (2) is in the form $y = a_0 e^{a_1 x}$, where $a_0 = y_0 e^{-a_1 x_0}$.

In our prognostic model we came out of the aluminium prices presented on LME. We dealt with the monthly averages of the daily closing aluminium prices "Cash Seller&Settlement price" in the period from December 2002 to June 2006. We obtained the market data from the official web page of the London Metal Exchange [6]. The course of the aluminium prices on LME (in US $ per tonne) in the observing period is presented in Fig. 1.

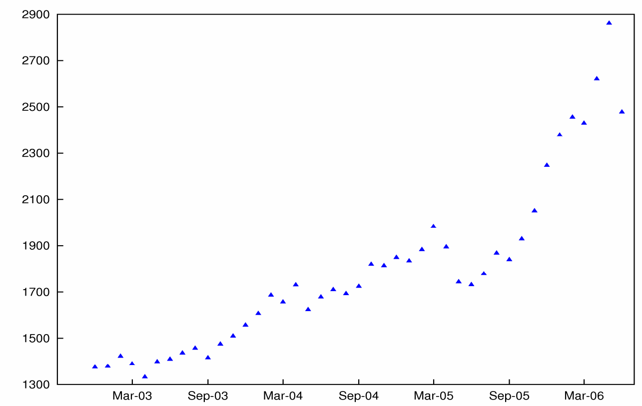

Fig.1 The course of the aluminium prices on LME in years 2003 - 2006

As we can see in Fig. 1 the course of the aluminium prices in the considered period changes markedly.

2. Mathematical model

We shall consider the Cauchy initial problem in the form (2)

$$y' = a_1 y$$

with the initial condition

$$y(x_0) = Y_0,$$

where $[x_0, Y_0]$ are the known values (x_0 is the order of the month, let $x_0 = 0$ and Y_0 is the aluminium price (stock exchange) on LME in the month x_0).

To determine the value of the unknown coefficient a_1, the second known point $[x_1, Y_1]$ is used, where $x_1 = 1$ and Y_1 is the aluminium price on LME in the month x_1. That means $[x_1, Y_1]$ are the values corresponding to the next month in comparison with those of $[x_0, Y_0]$.

Substituting the point $[x_1, Y_1]$ to the particular solution of the problem (2) we have

$$Y_1 = Y_0 e^{a_1 (x_1 - x_0)}.$$

After some manipulations we obtain the formula of the unknown coefficient a_1

$$a_1 = \frac{1}{x_1 - x_0} \ln\left(\frac{Y_1}{Y_0}\right).$$

Now we can substitute a_1 to the Cauchy initial problem (2) and we acquire

$$y' = \frac{1}{x_1 - x_0} \ln\left(\frac{Y_1}{Y_0}\right).y, \quad y(x_1) = Y_1.$$

Generalizing the previous principle we can get the Cauchy initial problem in the point x_i in the following form

$$y' = \frac{1}{x_i - x_{i-1}} \ln\left(\frac{Y_i}{Y_{i-1}}\right).y, \quad y(x_i) = Y_i, \quad i = 1, 2, 3, \dots . \qquad (3)$$

The unknown values of the aluminium prices are forecasted by the numerical solution of the Cauchy initial problem (3) using two chosen numerical methods:

- the method of exponential forecasting,
- the embedded Runge-Kutta formulae of the 5th order to the 6th order.

The method of exponential forecasting is based on the exponential aproximation of the solution of the Cauchy initial problem for the 1st order ordinary differential equations [4].

The method is using the following numerical formulae

$$x_{i+1} = x_i + h,$$

$$y_{i+1} = y_i + bh + Qe^{vx_i}\left(e^{vh} - 1\right),$$

for $i = 1, 2, 3, \dots$, where $h = x_{i+1} - x_i$ is the step of the constant size.

The unknown coefficients are calculated by means of these formulae

$$v = \frac{f''(x_i, y_i)}{f'(x_i, y_i)}, \quad Q = \frac{f'(x_i, y_i) - f''(x_i, y_i)}{(1-v)\, v^2 e^{v x_i}}, \quad b = f(x_i, y_i) - \frac{f'(x_i, y_i)}{v}.$$

If we consider the Cauchy initial problem (3), the function $f(x_i, y_i)$ has the form $f(x_i, y_i) = a_1 y_i$ and then $f'(x_i, y_i) = a_1 y'(x_i) = a_1{}^2 y_i$, $f''(x_i, y_i) = a_1{}^2 y'(x_i) = a_1{}^3 y_i$, where $a_1 = \dfrac{1}{x_i - x_{i-1}} \ln\left(\dfrac{Y_i}{Y_{i-1}}\right)$.

The embedded Runge-Kutta formulae of the 5th order to the 6th order are in the following forms [3]

$$x_{i+1} = x_i + h,$$

$$y_{i+1} = y_i + \frac{1}{144}\left(9 K_i^{[1]} + 40 K_i^{[3]} + 20 K_i^{[4]} + 30 K_i^{[5]} + 35 K_i^{[6]} + 10 K_i^{[7]}\right),$$

$$\hat{y}_{i+1} = \hat{y}_i + \frac{1}{288}\left(19 K_i^{[1]} + 75 K_i^{[3]} + 50 K_i^{[4]} + 50 K_i^{[5]} + 75 K_i^{[6]} - 9 K_i^{[7]} + 28 K_i^{[8]}\right),$$

where

$$K_i^{[1]} = h_i\ f(x_i, y_i),$$

$$K_i^{[2]} = h_i\ f(x_i + \frac{1}{10} h_i,\ y_i + \frac{1}{10} K_i^{[1]}),$$

$$K_i^{[3]} = h_i\ f(x_i + \frac{1}{5} h_i,\ y_i + \frac{1}{5} K_i^{[2]}),$$

$$K_i^{[4]} = h_i\ f(x_i + \frac{2}{5} h_i,\ y_i - \frac{1}{5} K_i^{[1]} + \frac{2}{5} K_i^{[2]} + \frac{1}{5} K_i^{[3]}),$$

$$K_i^{[5]} = h_i\ f(x_i + \frac{3}{5} h_i,\ y_i + \frac{31}{30} K_i^{[1]} - \frac{64}{30} K_i^{[2]} + \frac{43}{30} K_i^{[3]} + \frac{8}{30} K_i^{[4]}),$$

$$K_i^{[6]} = h_i\ f(x_i + \frac{4}{5} h_i,\ y_i + \frac{2}{35} K_i^{[1]} + \frac{20}{35} K_i^{[2]} + \frac{3}{35} K_i^{[3]} - \frac{34}{35} K_i^{[4]} + \frac{37}{35} K_i^{[5]}),$$

$$K_i^{[7]} = h_i\ f(x_i + h_i,\ y_i - \frac{20}{10} K_i^{[1]} + \frac{28}{10} K_i^{[2]} - \frac{18}{10} K_i^{[3]} + \frac{38}{10} K_i^{[4]} - \frac{25}{10} K_i^{[5]} + \frac{7}{10} K_i^{[6]}),$$

$$K_i^{[8]} = h_i\ f(x_i + h_i,\ y_i - \frac{9440}{5880} K_i^{[1]} + \frac{11342}{5880} K_i^{[2]} - \frac{9302}{5880} K_i^{[3]} + \frac{25982}{5880} K_i^{[4]} - \frac{17175}{5880} K_i^{[5]} + \frac{4473}{5880} K_i^{[6]}),$$

for $i = 1, 2, 3, \ldots$, where $h = x_{i+1} - x_i$ is the step of the constant size.

Using two known initial values $[x_{i-1}, Y_{i-1}]$ and $[x_i, Y_i]$ we calculate the prognosis y_{i+1} in the month x_{i+1}, $i = 1, 2, 3, \ldots$. In this way, using the known aluminium stock exchanges in December 2002 and January 2003, we gradually determine the price prognoses in the next months in the period from February 2003 to June 2006. Thus, we obtain the prognoses for 41 months. Each aluminium price is calculated by using a current form of the Cauchy initial problem (3).

Two types of forecasting are created:

- monthly forecasting
  Using the values $[x_{i-1}, Y_{i-1}]$ and $[x_i, Y_i]$, we directly obtain $[x_{i+1}, y_{i+1}]$ for $i = 1, 2, 3, \ldots, 40$.
- daily forecasting
  In this case from the values $[x_{i-1}, Y_{i-1}]$ and $[x_i, Y_i]$ the prognosis y_{i+1} in the month x_{i+1} is obtained using more partial computations.
  The interval $\langle x_i, x_{i+1} \rangle$ of the lenght $h = 1$ month is divided into n parts, where n is the number of the trading days on LME in the month x_{i+1}. We gain the sequence of the points $x_{i0} = x_i$, $x_{ij} = x_i + \frac{h}{n} j$, for $j = 1, 2, \ldots, n$, where $x_{in} = x_{i+1}$.
  For each point of the subdivision of the interval a current form of the Cauchy initial problem (3), which is solved by the chosen numerical methods, is created. In this way we obtain the prognoses of the aluminium price on the trading days y_{ij}. By computing the arithmetic mean of the daily prognoses we obtain the monthly prognosis of the aluminium price in the month x_{i+1}. Thus, $y_{i+1} = \frac{\sum_{j=1}^{n} y_{ij}}{n}$. This type of forecasting responds more to creating the real monthly averages of the daily closing aluminium prices on the London Metal Exchange. Consequently, we assume that this type of forecasting gives us more accurate prognoses than monthly forecasting.

The calculated prognoses are compared with the real stock exchanges. We evaluate the difference between them, which is denoted by $\delta_s = y_s - Y_s$ (prognosis deviation) and the ratio of the prognosis deviation from the real price, i.e. $p_s = \frac{\delta_s}{Y_s}.100\,\%$, $s = 2, 3, \ldots, 42$. The numerical computations are made by using programs written in Turbo Pascal.

3. Results

3.1 Comparing the accuracy of the chosen numerical methods

Using the different numerical methods to solve the Cauchy initial problem (3) we obtain the same results by daily forecasting and the different results by monthly forecasting. That is the reason why we analyse only the results of monthly forecasting to determine the success of the chosen numerical methods.

The numerical methods were compared by means of the following criterions:

- the number of more accurate price prognoses when comparing the two chosen numerical methods in the observing months,
- the number of the critical values of forecasting (it means the prognosis with the absolute percentage error exceeded 10 % of the real stock exchange),

- the arithmetic mean of all absolute percentage monthly deviations, $\frac{\sum_{s=2}^{42} |p_s|}{41}$,
- the distribution of the number of the prognoses according to their absolute percentage monthly deviation from the real price.

The obtained results are presented in Table 1 and Table 2.

Table 1 The comparison of the success of monthly forecasting by means of the different numerical methods

Criterion	The method of exponential forecasting	The embedded Runge-Kutta formulae of the 5th to the 6th order
The number of more accurate price prognoses	22	18
The number of the critical values	3	3
The arithmetic mean of all absolute percentage monthly deviations	4,65 %	4,64 %

Table 2 The distribution of the number of the prognoses according to their absolute percentage monthly deviation from the real price

The absolute percentage monthly deviation	The method of exponential forecasting	The embedded Runge-Kutta formulae of the 5th to the 6th order
< 5 %	27	27
$\langle 5\%, 7.5\%)$	8	7
$\langle 7.5\%, 10\%)$	3	4
$\geq 10\%$	3	3

If we compare the results of monthly forecasting by using the chosen numerical methods, the differences are minimal. The similar arithmetic mean of all absolute percentage monthly deviations, the number of more accurate prognoses (monthly forecasting aquired the same prognoses in 1 month) and the distribution of the number of the prognoses according to their absolute percentage monthly deviation from the real price point at this fact. The number of the critical values is the same.

When comparing the different forecasting results, we can say that monthly forecasting by using the two chosen numerical methods is similar in the stable trends in the course of the aluminium prices, as follows:

- the trend of the prices is increasing and the next prognosed price Y_{i+1} increases, too ($Y_{i-1} < Y_i < Y_{i+1}$),
- the trend of the prices is decreasing and the next prognosed price Y_{i+1} falls, too ($Y_{i-1} > Y_i > Y_{i+1}$).
- The differences in forecasting by using our numerical methods are denoted when the price trend is changed:
- the trend of the prices is increasing ($Y_{i-1} < Y_i$), but the following prognosed price Y_{i+1} falls,
- the trend of the prices is decreasing ($Y_{i-1} > Y_i$), but the following prognosed price Y_{i+1} increases.

In the case of the decline in the increasing trend forecasting by the embedded Runge-Kutta formulae of the 5th order to the 6th order is more accurate. But if the prognosed price increases in the decreasing trend, the method of exponential forecasting is more accurate.

The following Table 3 shows the number of more accurate prognoses by comparing the mentioned numerical methods according to the trends of the aluminium prices.

Table 3 The number of more accurate prognoses by comparing the mentioned numerical methods according to the trends of the aluminium prices on LME

The trend of the prices		The method of exponential forecasting	The embedded Runge-Kutta formulae of the 5th to the 6th order	Total
Stable trend	the trend of the prices is increasing and the prognosed price increases	9	7	16
	the trend of the prices is decreasing and the prognosed price falls	1	2	3
Table 3: continued from previous page				
Variable trend	the trend of the prices is increasing and the prognosed price falls	3	8	11
	the trend of the prices is decreasing and the prognosed price increases	9	1	10
Total		22	18	40

3.2 The analyze of the forecasting results

We compared the prognoses obtained by means of the chosen numerical methods and the aluminium stock exchange in the observing months according to the trends of the prices. We have found out that in the increasing price trend the prognosis calculated by the method of exponential forecasting is usually higher than the prognosis obtain by the embedded formulae.

- the stable increasing trend
  The higher increase of the prognosis aquired by the method of exponential forecasting is an advantage in the stable increasing trend if the increase of the price raises its intensity, i. e. $(Y_i - Y_{i-1}) < (Y_{i+1} - Y_i)$ (8 prognoses). In case of the decreasing intensity of the increase in the prices, i. e. $(Y_i - Y_{i-1}) > (Y_{i+1} - Y_i)$, the higher increase of the prognosis makes larger distance from the slower increasing real price. Thus, forecasting by the embedded Runge-Kutta formulae is more advantageous (5 prognoses). In the stable increasing trend we gained also 3 prognoses, where the increase of the prognosis obtained by the method of exponential forecasting is lower than the increase of the prognosis determined by the embedded formulae. Therefore in the rapid increase of the prices, the prognoses calculated by the embedded formulae are more accurate (2 prognoses) and in the slight increase of the prices, the prognosis obtained by the method of exponential forecasting is more accurate (1 prognosis).
- the increasing trend with the decline of the prognosed price
  The quicker increase of the prognosis obtained by the method of exponential forecasting is a disadvantage in this case. It is because of the larger distance of the prognosis from the real price, which is fallen. Thereby in this trend of the prices, forecasting by the embedded Runge-Kutta formulae of the 5th order to the 6th order is usually more accurate (8 prognoses). Only in 3 cases the increase of the prognosis obtained by the method of exponential forecasting is lower than the increase of the prognosis calculated by the embedded formulae. Thus, forecasting by the method of exponential forecasting is more accurate.

According to the acquired results, it can also be said that the prognoses determined by the embedded Runge-Kutta formulae of the 5th order to the 6th order usually fall more in the decreasing trend than the prognoses calculated by the method of exponential forecasting.

- the stable decreasing trend
  The mentioned larger decline of the prognosis obtained by the embedded formulae is an advantage in the rapid decreasing trend $|Y_i - Y_{i-1}| < |Y_{i+1} - Y_i|$ (2 prognoses). If the decline of the prices is slighter $|Y_i - Y_{i-1}| > |Y_{i+1} - Y_i|$, then forecasting by the method of exponential forecasting is more advantageous (1 prognosis).
- the decreasing trend with the increase of the prognosed price
  The larger decline of the prognoses calculated by the embeded formulae is a disadvantage in the period of the change from the decreasing trend to the increasing one. The higher decline makes the higher prognosis error according to the increased real price. By that means, in this trend of the prices we aquired 9 more accurate prognoses determined by the method of exponential forecasting against 1 more accurate prognosis obtained by the embedded formulae. (In this unusual case the prognosis calculated by the method of exponential forecasting is lower than the prognosis gained by the embedded formulae.)

4. Conclusion

There are either no differences in forecasting between using the two chosen numerical methods (daily forecasting) or, if there are any, they are little (monthly forecasting). The diferences are visible according to the different trends in the course of the aluminium prices. By monthly forecasting, using the method of exponential forecasting we obtain usually higher values of the prognoses than by using the embedded Runge-Kutta formulae of the 5th order to the 6th order. This case occurs in 33 calculated prognoses, the opposite case includes 7 prognoses, and 1 prognosis obtains the same values by using both numerical methods. We can say that the forecasting results acquired by using the different chosen numerical methods are similar and neither of the methods is more accurate in forecasting the aluminium prices markedly.

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