THE NEW HYPOTHESIS OF NORMAL STRESS DISTRIBUTION FOR CONTACT ARC IN ROLLING

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Abstract
There is one subsistent group of relations based on theoretical knowledge which are used for the average contact pressure calculation on smooth rolls. The second group of relations is generated by empirical equations valid in restricted area of application. The new hypothesis of contact pressure distribution in dependence on rolling conditions is presented in this submission. A new term of non-dimensional value has been defined as relative stress. An equation for the average contact pressure calculation, presented by author moves the mathematical theory of rolling closer to practice. The relative stress is dependant on ratio l_d/h_{av}, reduction intensity and friction coefficient. The equation of relative stress in average contact pressure calculation allows for the presence of a minimum. The verification of new hypothesis of contact pressure distribution is in good consent with data measured in rolling practice.

Keywords: lengthwise rolling, differential equations, normal stress, relative stress

1 Introduction

A derivation of basic differential equation of power balance in a rolling zone for flat rollers is possibly found in common books dealing with rolling as Mielnik [1], Kollerová [2] and Pernis [3]. The geometry of zone in rolling is marked on **Fig.1**; simultaneously in general position an element is drawn and dimensioned. The stress impacting on this element is drawn on **Fig.2**. Description of quotation stated on **Fig.1** and **Fig.2** are following:

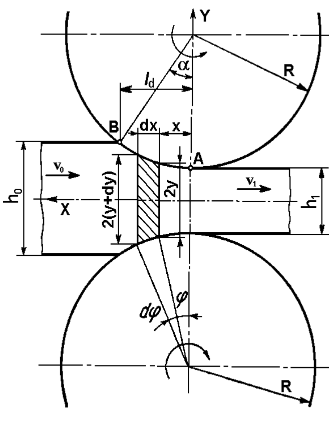

Fig.1 Determination of geometric relationships

σ_n – normal contact stress
τ – shear stress
σ_x , σ_y – principal stress (σ_3, σ_1)
σ_a – actual resistance to deformation
x ,y – element coordinates
dx, dy – coordinate differentials x and y
α – gripping angle
α_n – angle of neutral point
l_d – length of contact arc
h_0 , h_1 – thickness before and after deformation
Δh – absolute reduction

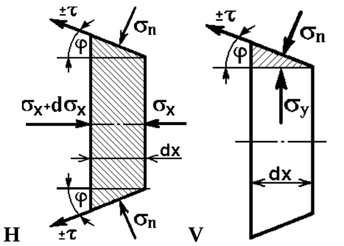

Fig.2 Determination of stresses acting on the element: H-horizontal, V-vertical

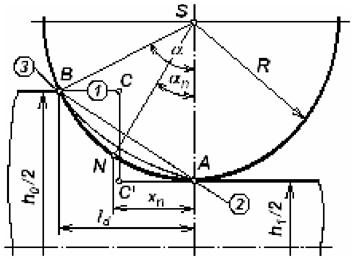

Fig.3 The contact Arc Approximation: 1- poly line,

ε – relative reduction
f – friction coefficient
R – radius of rollers

Based on forces affecting the selected element, a basic differential equation of roller pressure is derived for the ideal state with allowance for the forward and backward slip. The equation of plasticity used is $\sigma_n - \sigma_x = \sigma_a$, while implementing the assumption of principal stress $\sigma_y \doteq \sigma_n$ and shear stress $\tau = f \cdot \sigma_n$. Equation describing stresses relationship in rolling gap has a form:

$$\frac{d\sigma_n}{dx} - \frac{\sigma_a}{y} \cdot \frac{dy}{dx} \pm \frac{f}{y} \cdot \sigma_n = 0$$
(1)

General solution of differential eq. (1) is mentioned by Brzobohatý [4] in the form

$$\sigma_n = e^{\pm\int \frac{f}{y} \cdot dx} \cdot \left(C + \int \frac{\sigma_a}{y} \cdot e^{\pm\int \frac{f}{y} \cdot dx} \cdot dy \right) \tag{2}$$

Where C represents integration constant, which needs to be defined from boundary conditions. The contact arc is mathematically described by equation of a circle. After circle equation nomination and its differentials into the differential eq. (1), remains only as a function of coordinate x alone with direct stress, that is $\sigma_n' = f(\sigma_n, x)$. The solution of such differential equation is analytically unknown. In the literature is possible to find more solutions of the differential eq. (1), where authors establish in the equation certain simplifications. The simplest solution was introduced by Korolev [5], while the contact arc (BA) is substituted by poly line (BCC'A), see **Fig.3** – curve 1. A more particular solution was introduced by Tselikov [6]. Contact arc (BA) has been substituted by straight line (BA), **Fig.3** – curve 2. However, implemented simplifications can not permit precise description of normal contact stress distribution acquired by measuring forces on rolling mills.

2 Solution of differential equation

The aim of this subscription is to further approximate the mathematical theory of rolling with practice. The solution reposes in substitution of circle arc (BA) meeting rolling material by the parabola (BA), **Fig.3** curve – 3, allowing the eq. (1) to be integrated [7]. The parabola equation is constructed in the way that it runs through point B and has vertex in point A.

$$y = ax^2 + b \tag{3}$$

where

$$a = \frac{\Delta h}{2 \cdot l_d^2} \quad , \quad b = \frac{h_1}{2} \tag{4), (5}$$

Tselikov [6] substituted length variable *x*, by angular variable *u* according to substitution

$$x = \sqrt{\frac{b}{a}} \cdot tg\, u \tag{6}$$

Parabola eq. (3) after application of new variable *u* assumes the form

$$y = b \cdot \left(tg^2 u + 1\right) \tag{7}$$

Solution of differential eq. (2) needs definition of differential *dx* and ratio *dx/y*

$$dx = \sqrt{\frac{b}{a}} \cdot \frac{du}{cos^2 u} \quad , \quad \frac{dx}{y} = \frac{du}{\sqrt{ab}} \qquad (8), (9)$$

Calculation of first integral for eq. (2)

$$f \cdot \int \frac{dx}{y} = \frac{f}{\sqrt{ab}} \int du = m \cdot u \tag{10}$$

Where constant *m* is defined from relation

$$m = \frac{2f \cdot l_d}{\sqrt{\Delta h \cdot h_1}} \tag{11}$$

For second integral is needed to define differential *dy* and ratio *dy/y*

$$dy = 2b \cdot \frac{tg\, u}{cos^2 u} \cdot du \quad , \quad \frac{dy}{y} = 2tg\, u \cdot du \qquad (12), (13)$$

Transformation of second integral on variable *u*

$$\int \frac{\sigma_a}{y} e^{\int \frac{f}{y} \cdot dx} \cdot dy = 2\sigma_a \int e^{mu} \cdot tg\, u \cdot du \tag{14}$$

After establishing of a new variable *u* into second integral in eq. (2) a form is acquired, which according to [8] was given by Tselikov

$$\sigma_n = e^{\pm mu} \left(C + 2\sigma_a \cdot \int e^{\mp mu} \cdot tg\, u \cdot du\right) \tag{15}$$

where

$$u = arctg\left(\sqrt{\frac{\Delta h}{h_1}} \cdot \frac{x}{l_d}\right) \quad \text{that is} \quad u = arctg\left(\sqrt{\frac{\varepsilon}{1 - \varepsilon}} \cdot \frac{x}{l_d}\right) \tag{16}$$

Fraction under root represents relative deformation ε. In eq. (15) top signs (+, –) in exponential functions are valid for zone of forward slip and bottom signs (–, +) are valid for zone of backward slip. In eq. (15) it is needed to determine the function integral e^{mu}.*tg u*. This integral can be determined under the assumption $tg\,u \doteq u$. This inaccuracy can be allowed for in small and medium deformation values. For zone of backward slip, eq. (15) after reduction will have the form

$$\sigma_{nB} = e^{-mu}\left(C_B + 2\sigma_a \int e^{mu} \cdot u \cdot du\right) \tag{17}$$

After integral calculation

$$\sigma_{nB} = e^{-mu}\left[C_B + 2\sigma_a \cdot e^{mu} \cdot \left(\frac{u}{m} - \frac{1}{m^2}\right)\right] \tag{18}$$

where: C_B is an integral constant, which is for the zone of backward slip determined from conditions in point B (see **Fig.1**). For $x=l_d$ from equation of plasticity $\sigma_{nB} - \sigma_x = \sigma_a$, where $\sigma_x=0$ (without backward tension), results $\sigma_{nB}=\sigma_a$. Value $u=u_0$ for $x=l_d$ results from eq. (16)

$$u_0 = arctg\left(\sqrt{\frac{\varepsilon}{1-\varepsilon}}\right) \tag{19}$$

After substituting of boundary conditions in point B into eq. (18) integral constant is determined

$$C_B = 2\sigma_a\left[\frac{1}{2} - \left(\frac{u_0}{m} - \frac{1}{m^2}\right)\right]e^{mu_0} \tag{20}$$

After substituting of integral constant C_B into eq. (18) is determined an equation for the process contact stress σ_{nB} calculation in the zone of backward slip

$$\sigma_{nB} = 2\sigma_a\left\{\left[\frac{1}{2} - \left(\frac{u_0}{m} - \frac{1}{m^2}\right)\right]e^{m(u_0-u)} + \left(\frac{u}{m} - \frac{1}{m^2}\right)\right\} \tag{21}$$

For the zone of forward slip will be used the equation

$$\sigma_{nF} = e^{mu}\left(C_F + 2\sigma_a \int e^{-mu} \cdot u \cdot du\right) \tag{22}$$

After integration

$$\sigma_{nF} = e^{mu}\left[C_F - 2\sigma_a \cdot e^{-mu} \cdot \left(\frac{u}{m} + \frac{1}{m^2}\right)\right] \tag{23}$$

where C_F is integral constant, which is for the zone of forward slip determined by conditions in point A (see **Fig.1**). For $x=0$ from equation of plasticity $\sigma_{nF} - \sigma_x = \sigma_a$, where $\sigma_x=0$ (without forward tension), results $\sigma_{nF}=\sigma_a$. Value $u=0$ for $x=0$ results from eq. (16)

$$C_F = 2\sigma_a\left(\frac{1}{2} + \frac{1}{m^2}\right) \tag{24}$$

After substituting of integral constant C_F into eq. (23) is determined an equation for the process contact stress σ_{nF} calculation in the zone of forward slip

$$\sigma_{nF} = 2\sigma_a\left[\left(\frac{1}{2}+\frac{1}{m^2}\right)e^{mu} - \left(\frac{u}{m}+\frac{1}{m^2}\right)\right] \tag{25}$$

By this equation in Tselikov terminates his interpretation regarding solution of differential eq. (1) and contact pressure calculation. From literature, the calculation of average contact pressure $\sigma_{n,av}$ is not known from equations (21) and (25).

3 Neutral section

Based on present observations, the author submits a deduction of equation for the average contact pressure $\sigma_{n,av}$ calculation, which requires position determination of a neutral point. In general, the neutral point N (see **Fig.3**) is determined by comparison of right sides of equations (21) and (25). Value $u=u_n$ is in this point determined by section margin in the zone of backward and forward slip (neutral point)

$$\sigma_{nB}(u_n) = \sigma_{nF}(u_n) \tag{26}$$

Solitary coordinate x_n in these equations represents the variable u_n, according to eq. (16)

$$u_n = arctg\left(\sqrt{\frac{\varepsilon}{1-\varepsilon}}\cdot\frac{x_n}{l_d}\right) \quad , \qquad x_n = \sqrt{\frac{1-\varepsilon}{\varepsilon}}\cdot l_d \cdot tg\, u_n \tag{27}$$

The particular comparison of right sides of equations (21) a (22) results in transcendental equation dissolution

$$2\sigma_a\left\{\left[\frac{1}{2}-\left(\frac{u_0}{m}-\frac{1}{m^2}\right)\right]e^{m(u_0-u_n)}+\left(\frac{u_n}{m}-\frac{1}{m^2}\right)\right\} = 2\sigma_a\left[\left(\frac{1}{2}+\frac{1}{m^2}\right)e^{mu_n} - \left(\frac{u_n}{m}+\frac{1}{m^2}\right)\right] \tag{28}$$

From this equation it is needed to express the variable u_n. By modification and gradual reduction of equation in a way that constants are concentric into brackets

$$\left(0{,}5m^2 - mu_0 + 1\right)\cdot e^{m(u_0-u_n)} - \left(0{,}5m^2+1\right)\cdot e^{mu_n} + 2mu_n = 0 \tag{29}$$

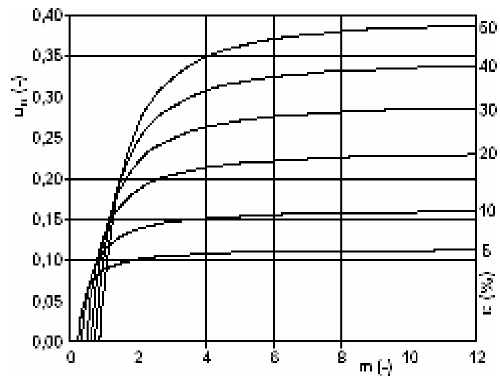

Fig.4 Dependence of the neutral value u_n on constant m and deformation

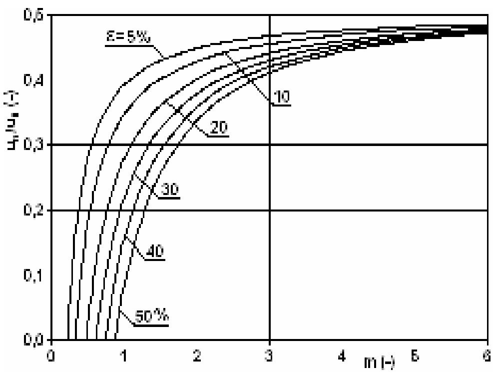

Fig.5 Convergence ratio u_n/u_0 to the value 0.5

Solitary value u_n is expressed by explicit function $F(m,\varepsilon,u_n)=0$. Therefore, to determine the value u_n , a graph given on **Fig.4**. has been worked out by numerical methods. A ratio of neutral value u_n to value u_0 with the value growth *m* converges to 0.5, see **Fig.5**

$$\lim_{m\to\infty}\frac{u_n}{u_0}=0.5 \tag{30}$$

4 Average normal contact stress

Representation of the contact pressure process along the contact act is presented on **Fig.6**. Average normal contact stress $\sigma_{n,av}$ is determined as average integral value of variable *u* in the interval $u \in <0;u_0>$. The calculation needs to be implemented independently for the zone of backward and forward slip. Zone of forward slip $\sigma_{nF}(u)$ eq. (25), for *u* in the interval $u \in <0;u_n>$. Zone of backward slip $\sigma_{nB}(u)$ eq. (21), for *u* in the interval $u \in <u_n;u_0>$

$$\sigma_{n,av}=\frac{1}{u_0}\cdot\int_0^{u_0}\sigma_n(u)\cdot du=\frac{1}{u_0}\cdot\left[\int_0^{u_n}\sigma_{nF}(u)\cdot du+\int_{u_n}^{u_0}\sigma_{nB}(u)\cdot du\right]=\frac{1}{u_0}\cdot\left[I_F+I_B\right] \tag{31}$$

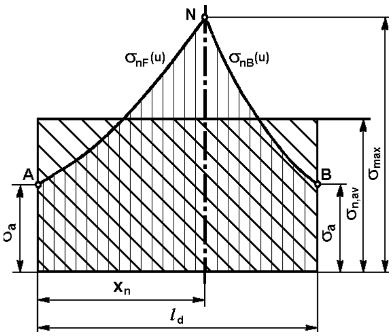

Fig.6 Average normal contact stress n,av

An integration boundary u_n represents a value which determines the position of a neutral point and is defined by the explicit eq. (29). The value of integration boundary u_0 is given by the eq. (19). For eq. (31) we will determine integrals I_F a I_B

$$I_F=\int_0^{u_n}\sigma_{nF}(u)\cdot du=\sigma_a\cdot\frac{2}{m}\left[\left(\frac{1}{2}+\frac{1}{m^2}\right)e^{mu_n}-\left(\frac{u_n}{m}+\frac{1}{m^2}\right)-\frac{1}{2}\left(u_n^2+1\right)\right] \tag{32}$$

$$I_B=\int_{u_n}^{u_0}\sigma_{nB}(u)\cdot du=\sigma_a\cdot\frac{2}{m}\left\{\left[\frac{1}{2}-\left(\frac{u_0}{m}-\frac{1}{m^2}\right)\right]e^{m(u_0-u_n)}+\left(\frac{u_n}{m}-\frac{1}{m^2}\right)+\frac{1}{2}\left(u_0^2-u_n^2-1\right)\right\} \tag{33}$$

After substituting equations (32) and (33) into eq. (31) and a mathematical modification we acquire an equation which describes the average normal contact stress $\sigma_{n,av}$

$$\sigma_{n,av}=\sigma_a\frac{2}{mu_0}\left[\left(1+\frac{2}{m^2}\right)\cdot\left(e^{mu_n}-1\right)+\frac{u_0^2}{2}-u_n\left(u_n+\frac{2}{m}\right)\right] \tag{34}$$

For practical use is the eq. (34) expressed as a relative stress $\overline{\sigma}$, which states ratio of the average normal stress $\sigma_{n,av}$ to actual deformation resistance σ_a that is $\overline{\sigma} = \sigma_{n,av}/\sigma_a$. The determination of actual deformation resistance is described in [9-11].

$$\overline{\sigma} = \frac{2}{mu_0} \cdot \left[\left(1 + \frac{2}{m^2} \right) \cdot \left(e^{mu_n} - 1 \right) + \frac{u_0^2}{2} - u_n \cdot \left(u_n + \frac{2}{m} \right) \right] \tag{35}$$

A visualization of the eq. (35) presented in graph on **Fig.7**. Curves in zone $0<m<1$ are not well readable, therefore on **Fig.8** is given an expansion of this zone $\overline{\sigma}-$function, eq. (35) is in the point $m=0$ intermittent. The visualization is implemented in dependence on value *m*, while the

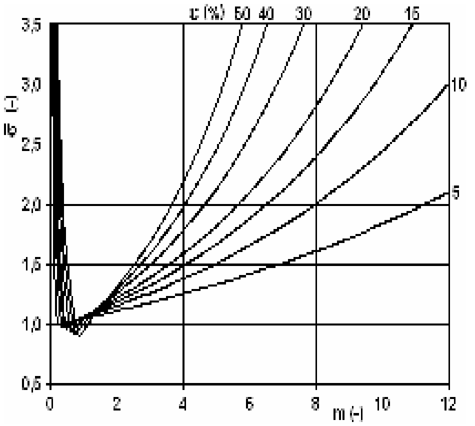

Fig.7 Dependence of the neutral value u_n on constant *m* and deformation

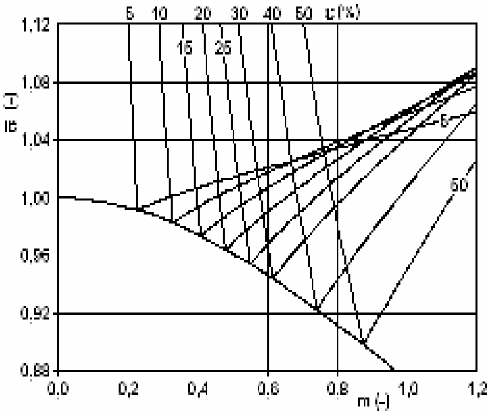

Fig.8 Convergence ratio u_n/u_0 to the value 0.5

relative reduction ε is a parameter. As can be seen from **Fig.8**, $\overline{\sigma}-$function has a local minimum, what is in accordance with practical measurements of average contact pressure. With a reduction growth the position of local minimum $\overline{\sigma}-$function decreases and moves towards higher values *m*. More detailed description of properties of $\overline{\sigma}-$function is presented in [12,13]. Empirical equations describing $\overline{\sigma}-$function are well submitted by Hajduk and Konvičný [14]. Empirical functions of relative stress, that means of $\overline{\sigma}-$function are designed in dependence $\overline{\sigma} = \overline{\sigma}(l_d/h_{av},\varepsilon)$, however most often in form $\overline{\sigma} = \overline{\sigma}(l_d/h_{av})$. Such dependence is described by Rusz et all [15], where measured data account for the local minimum, proved by eq. (35). Validity verification of the eq. (34) has been implemented on measured data from publication Kvačkaj et all [16]. Under cold rolling sheet has been rolled with the entry thickness of h_0=0.65 mm, on one pass. The material was dynamo steel C-Si. The roll diameter 210 mm. Circumferential velocity 0.66 m/s. Dependence of measured and calculated results according to eq. (34) in dependence on ratio l_d/h_{av} is presented on **Fig.9**. The graph is simultaneously for comparison completed with values of the average normal stress calculated according to Korolev, Tselikov and author (eq. 34). The calculated values of contact pressure according to Tselikov are by 50 % lower than the measured [16] in gross proportion of ratio l_d/h_{av}. Contact pressure calculated according to equation (34) and Korolev has a faster tendency of growth than the actual measured values in [16]. All theoretical calculations of contact pressure are strongly dependant on correct estimation of friction coefficient. For the calculation of contact pressure in given proportion of ratio l_d/h_{av} on **Fig.9** is more suitable to use the equation (34) or according to Korolev.

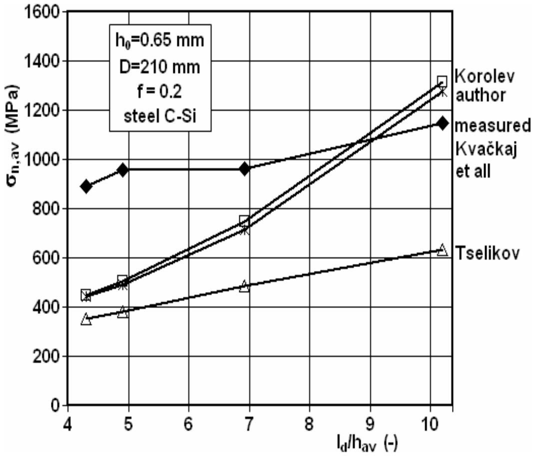

Fig.9 Average contact stress $\sigma_{n,av}$ measured and calculated

5 Conclusion

Present theories, describing calculation of the average normal stress are based on certain simplified assumptions. This causes significant differences between the theory and measured data. The submitted contribution increases concurrence of mathematical theory of rolling with the measured values of relative stress. Main contribution of the new hypothesis of the division of contact pressure rests in the fact that it theoretically proves the existence of the local minimum in the average contact pressure. The minimum is not a constant point, yet it is dependant on relative deformation intensity. Equation (35), which describes the relative stress, is function in the form $\overline{\sigma} = \overline{\sigma}(l_d/h_{av}, \varepsilon, f)$. In comparison with empirical functions, $\overline{\sigma}$ – function allows also for the friction coefficient impact on relative stress. Tselikov in [6] for the calculation of relative stress at hot rolling in dependence on ratio l_d/h_{av} suggested 3 equations. Equation (35) solves the calculation of the average contact pressure in complex measure of ratio l_d/h_{av}. The report brings new knowledge in the area of mathematical theory of rolling.

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