DEFORMATION DISTRIBUTION IN ROLLING ZONE

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Received 27.07.2011
Accepted 13.09.2011

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Abstract
The main aim of the present paper is the distribution of relative deformation in rolling zone; due to the fact that relative deformation along the length of contact arc is changed. Mathematical analysis of the distribution of relative deformation covers elliptic distribution. Average value of relative deformation in rolling zone was obtained as integral value which more accurately describes total quantity of deformation. The present result shows that average value of relative deformation in rolling zone is not equal to total deformation for one pass, that means is possible to take into consideration for invariable equal to 2/3. Invariable of rolling zone equal to 2/3 is usable to evaluate resistance deformation from strain hardening curve by cold rolling. Calculated resistance deformation through rolling invariable more accurately describes actual value of resistance deformation.

Keywords: rolling, relative deformation, deformation distribution, rolling invariable, resistance deformation

1 Introduction
It is well known that simulation analysis, including mathematical and numerical simulation, is a proven and reliable technique for analyzing various forming processes [1-4]. Mathematical analysis helps understanding the deformation complexity of rolling process, especially during cold rolling.
In the rolling process the size of achieved deformation is evaluated through relative deformation [5, 6]. The value of deformation is evaluated for one pass through rolling mill [7, 8]. However, this value does not provide immediate image of deformation in rolling zone. Material in the moment of input into rollers is not deformed yet, but it is fully deformed at the output from the rollers [9, 10]. Deformation distribution on the length of contact arc has serious influence on the calculation of resistance deformation during cold rolling (strain-hardening of material) [11, 12]. General insight on longitudinal rolling process is stated in [13, 14].
Submitted analysis of relative deformation distribution in rolling zone is looking for an answer on immediate deformation distribution in the steady-state of rolling process.

2 Deformation in rolling zone
Geometrical characteristic of rolling zone is shown in **Fig. 1**. Total relative deformation ε obtained for one pass of material through rolling mill is given

$$\varepsilon = \frac{h_0 - h_1}{h_0} \tag{1}$$

where h_0 is thickness of material at the input into rollers and h_1 is thickness of material at the output from rollers. However, the equation (1) does not describe distribution of deformation in the rolling zone. Point E in rolling zone (see **Fig. 1**) is characterized by distance x from origin of coordinates and thickness h_x. Then, relative deformation ε_x of rolled material after pass from point B to point E is given by formula

$$\varepsilon_x = \frac{h_0 - h_x}{h_0} \tag{2}$$

where thickness h_x is established by equation of an circle with diameter *2R* and displacement y_0 in Y direction within the distance y_0

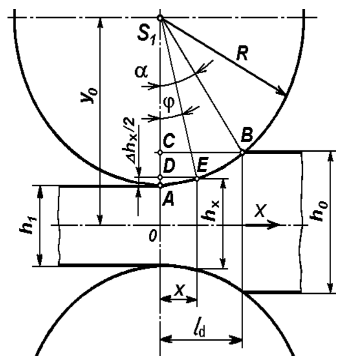


Fig.1 Geometry of rolling zone

$$x^2 + (y - y_0)^2 = R^2 \tag{3}$$

Thickness h_x is equal to double of Y coordinate in point E

$$h_x = 2\left(y_0 - \sqrt{R^2 - x^2}\right) \tag{4}$$

Displacement of circle centre y_0 is equal to the sum of roller radius *R* and half thickness of material at the output $h_1/2$

$$y_0 = R + \frac{h_1}{2} \tag{5}$$

Substituting the equations (4) a (5) into the equation (2) gives the relation

$$\varepsilon_x = \frac{h_0 - h_1 - 2R + 2\sqrt{R^2 - x^2}}{h_0} \tag{6}$$

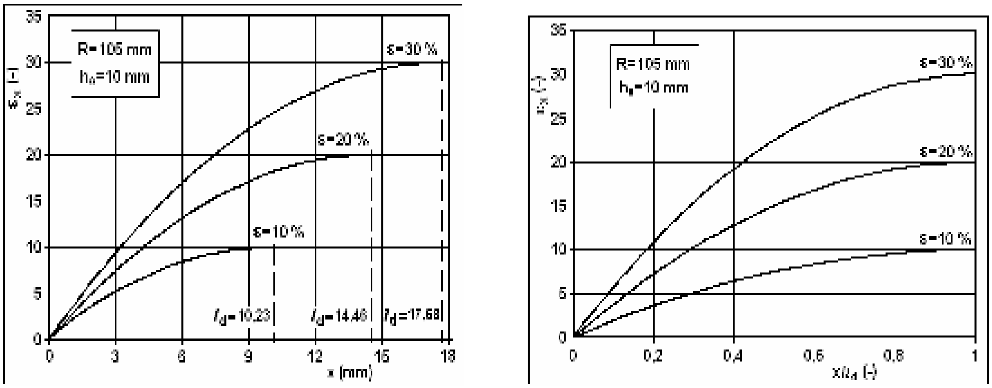
It can be seen that total relative deformation ε (see equation (1)) is involved in the equation (6). By introducing the substitution ε into the equation (6) and after retreatment we obtain the equation which describes distribution of relative deformation in rolling zone along the length of contact arc l_d

$$\varepsilon_x = \varepsilon - 2 \cdot \frac{R}{h_0} \cdot \left(1 - \sqrt{1 - \left(\frac{x}{R}\right)^2}\right) \tag{7}$$

Coordinate x belongs to the interval [0, l_d]. The length of contact arc l_d can be obtained from formula

$$\boldsymbol{l_d} = \sqrt{R \cdot \Delta h - \frac{\Delta h^2}{4}} \tag{8}$$

Graphs of distribution of relative deformation in rolling zone are showed in **Fig. 2**. Length of contact arc in absolute length scale is given in **Fig. 2a** (see equation (7)). Point A in **Fig. 1** corresponds to value x=0 and point B corresponds to coordinate $x=l_d$. Image of point B is moving and depends on the length of rolling zone. Distribution of relative deformation in rolling zone in dependence on normalized coordinate x along length of contact arc x/l_d is showed in **Fig. 2b**. This coordinate moves in interval $x/l_d \in <0;1>$. In this case relative deformation corresponding to point B lies on coordinate x/l_d=1 for all values of deformation. Graph showed in **Fig. 2b** corresponds to equation (9)



a) b)

Fig.2 Behavior of deformation ε_x along length of contact arc: a) in absolute length scale b) in relative scale (normalized on length of contact arc)

$$\varepsilon_x = \varepsilon - 2\cdot\frac{R}{h_0}\cdot\left(1-\sqrt{1-\left(\frac{l_d}{R}\right)^2\cdot\left(\frac{x}{l_d}\right)^2}\right) \tag{9}$$

The equation (7) defines the distribution of relative deformation in rolling zone. After modification the equation can be expressed as follows

$$\frac{x^2}{R^2}+\frac{\left(\varepsilon_x-\varepsilon+\frac{2R}{h_0}\right)^2}{\left(\frac{2R}{h_0}\right)^2}=1 \tag{10}$$

It stands to reason that eq. (10) is equation of an ellipse. It means that distribution of relative deformation in rolling zone governs elliptic distribution with the semi-major axis $a=R$ and the semi-minor axis $b=2R/h_0$. Translation of centre of the ellipse in the Y direction is $\varepsilon_{x0}=\varepsilon-2R/h_0$. It results from **Fig. 2** that relative deformation is changed along length of contact arc.

3 Average value of relative deformation

Average value of relative deformation ε_{av} is used to evaluate immediate value of deformation in rolling zone. Geometric characteristic of average value of relative deformation is showed in **Fig. 3**. Average value of relative deformation is given by formula

$$\varepsilon_{av} = \frac{1}{l_d}\int_0^{l_d} \varepsilon_x \cdot dx \tag{11}$$

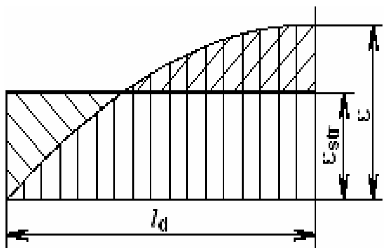


Fig.3 Average value of relative deformation in rolling zone

Substituting the equation (7) into the equation (11) gives the relation

$$\varepsilon_{av} = \frac{1}{l_d}\int_0^{l_d} \left\{ \varepsilon - 2\cdot\frac{R}{h_0}\cdot\left(1-\sqrt{1-\left(\frac{x}{R}\right)^2}\right)\right\}\cdot dx \tag{12}$$

In order to integrate the equation it is necessary to write equation as a sum of separate additive terms

$$\varepsilon_{av} = \frac{1}{l_d}\int_0^{l_d}\varepsilon dx - \frac{2R}{h_0\cdot l_d}\int_0^{l_d} dx + \frac{2R}{h_0\cdot l_d}\int_0^{l_d}\sqrt{1-\left(\frac{x}{R}\right)^2}\cdot dx \tag{13}$$

Substitution technique is used to simplify an integral of third term before evaluating it. We introduce a new variable *sin t* and differential *dx* as follows

$$\sin t = \frac{x}{R}\ , \qquad dx = R\cdot\cos t\cdot dt \qquad (14), (15)$$

The limits of integration must be written also in terms of new variable *t* as

$$t_1=0 \text{ and } t_2 = \arcsin\left(\frac{l_d}{R}\right) \qquad (16), (17)$$

Based on **Fig. 1** fraction l_d/R indicates sinus of gripping angle α so that the upper limit of integration $t_2=\alpha$ (replacing of fraction R/l_d with relation $1/\sin\alpha$). Then, the equation for the calculation of average relative deformation in rolling zone can be written

$$\varepsilon_{av} = \varepsilon + \frac{2R}{h_0}\left(\frac{1}{2}\cos\alpha + \frac{1}{2}\cdot\frac{\alpha}{\sin\alpha} - 1\right) \tag{18}$$

During cold rolling the strain hardening of material occurs and therefore it is necessary to establish a function of deformation resistance on relative deformation. For this purpose it is appropriate to evaluate ratio of average deformation to total relative deformation $\varepsilon_{av}/\varepsilon$

$$\frac{\varepsilon_{av}}{\varepsilon} = 1 + \frac{2R}{\varepsilon h_0}\left(\frac{1}{2}\cos\alpha + \frac{1}{2}\cdot\frac{\alpha}{\sin\alpha} - 1\right) \tag{19}$$

Term εh_0 presents absolute deformation Δh. Formula for gripping angle can by written based on rectangular triangle S_1CB in **Fig. 1** as follows

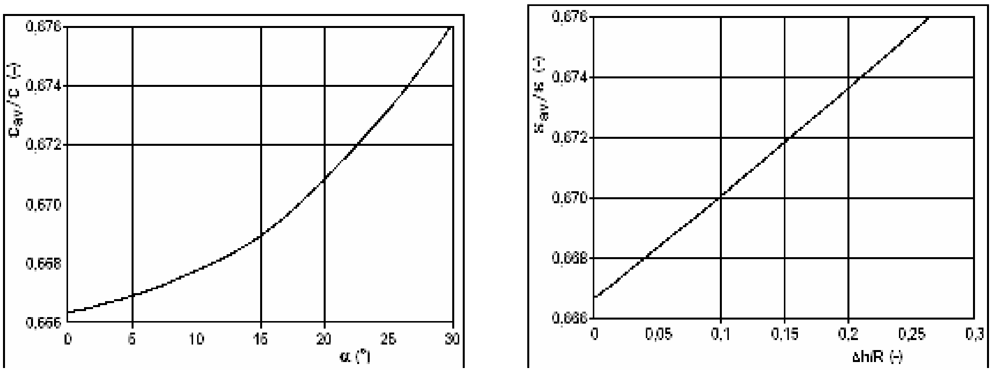
$$\cos\alpha = 1 - \frac{\Delta h}{2R} \tag{20}$$

Substituting the equation (20) into the equation (19) we obtain functional dependence of $\varepsilon_{av}/\varepsilon$ ratio on gripping angle

$$\frac{\varepsilon_{av}}{\varepsilon} = \frac{1}{2(1-\cos\alpha)} \cdot \left(\frac{\alpha}{\sin\alpha} - \cos\alpha \right) \tag{21}$$

4 Invariable of rolling zone

As functional dependence is expressed by the relation $\varepsilon_{av}/\varepsilon = f_1(\alpha)$ it is also possible, considering equation (20), to express functional dependence as follows $\varepsilon_{av}/\varepsilon = f_2(\Delta h/R)$. Visualization of both functional dependencies is given in **Fig. 4**. It can be seen from **Fig. 4** that $\varepsilon_{av}/\varepsilon$ ratio in rolling zone moves just within narrow interval $\varepsilon_{av}/\varepsilon \in < 0{,}666; 0{,}676 >$. Investigation of this ratio for gripping angle α>30° is practically useless because if gripping angle corresponded to the coefficient of friction it would get the value of $f=tg(\alpha)$=0,577.



a, b,

Fig.4 Dependency ratio of the mean deformation to total deformation $\varepsilon_{av}/\varepsilon$ in the roll gap: a, contact arc b, ratio Δh/R

The value of friction coefficient between material and rollers is too high, what can be obtained by roughening the rollers. As can be seen in **Fig. 4a** curve of $\varepsilon_{av}/\varepsilon$ ratio reaches at α=0 limit value. It is not possible to calculate the value directly from the equation (21) because at point α=0 the function is discrete. Limit value of $\varepsilon_{av}/\varepsilon$ ratio can be determined through limit at that point. It has been proved by double use of L'Hopital's rule that limit converges [15]

$$\lim_{\alpha \to 0} \frac{1}{2(1-\cos\alpha)} \cdot \left(\frac{\alpha}{\sin\alpha} - \cos\alpha \right) = \frac{2}{3} \tag{22}$$

For practical purposes it is possible to consider $\varepsilon_{av}/\varepsilon$ ratio to be invariable in rolling zone

$$\frac{\varepsilon_{av}}{\varepsilon} = \frac{2}{3} \tag{23}$$

5 Application of rolling zone invariable 2/3

The use of equation (23) is explained in the following example. Suppose that we use exponential model of strain hardening curve to determine deformation resistance, i.e.

$$\sigma_p = \sigma_0 + k \cdot \varepsilon^n \tag{24}$$

where σ_p is resistance deformation (MPa), ε represents relative deformation (%) and σ_0, k as well as n are material constants. Determination of actual resistance deformation is given in [16] and [17]. During cold rolling process the strain hardening of material with relative deformation ε_1 is given by the equation (24). State of material before rolling is given by $\sigma_{p0}=\sigma_0$. After rolling the material is strain hardened in the state given by formula $\sigma_{p1} = \sigma_0 + k \cdot \varepsilon_1^n$. Resistance deformation $\sigma_{p\varepsilon}$ is established as average value (before and after rolling)

$$\sigma_{p\varepsilon} = \frac{\sigma_{p0} + \sigma_{p1}}{2} \tag{25}$$

By substituting particular values into the equation we get

$$\sigma_{p\varepsilon 1} = \sigma_0 + \frac{1}{2} k \cdot \varepsilon_1^n \tag{26}$$

When taking into account the equation (23), the relative deformation in rolling zone is established by average value

$$\varepsilon_{av} = \frac{2}{3} \varepsilon_1 \tag{27}$$

After substituting average value of relative deformation in rolling zone into strain hardening equation (24) we get

$$\sigma_{p\varepsilon 1_{av}} = \sigma_0 + \left(\frac{2}{3}\right)^n \cdot k \cdot \varepsilon_1^n \tag{28}$$

By comparing strain hardening increments resulting from equations (26) and (28) we get

$$\Delta\sigma_{p\varepsilon 1} = \sigma_{p\varepsilon 1} - \sigma_0 = \frac{1}{2} \cdot k \cdot \varepsilon_1^n \tag{29}$$

$$\Delta\sigma_{p\varepsilon 1_{av}} = \sigma_{p\varepsilon 1_{av}} - \sigma_0 = \left(\frac{2}{3}\right)^n \cdot k \cdot \varepsilon_1^n \tag{30}$$

$\Delta\sigma_{p\varepsilon 1_{av}} / \Delta\sigma_{p\varepsilon 1}$ ratio of increment is given by the equation (31), where resistance deformation increment $\Delta\sigma_{p\varepsilon 1_{av}}$ is calculated from average value of deformation ε_{1av} and resistance deformation increment $\Delta\sigma_{p\varepsilon 1}$ is calculated from average value. Examples of calculated coefficients k_σ are given in **Table 1**.

Table 1 The influence of strain hardening coefficient n on ratio of strain hardening increment

n	0,25	0,5	0,75	1
$\frac{\Delta\sigma_{p\varepsilon 1_{av}}}{\Delta\sigma_{p\varepsilon 1}}$	$2 \cdot \left(\frac{2}{3}\right)^{0,25}$	$2 \cdot \left(\frac{2}{3}\right)^{0,5}$	$2 \cdot \left(\frac{2}{3}\right)^{0,75}$	$2 \cdot \left(\frac{2}{3}\right)^{1} = \frac{4}{3}$
value of coefficient k_σ	1,807	1,633	1,476	1,333

For material constant n=1 it is linear model of strain hardening, and ratio of increments reaches value k_σ=1.333. With the decreasing value of material constant *n*, the value of ratio of strain hardening in rolling zone is increasing.

$$k_\sigma = \frac{\Delta\sigma_{p\varepsilon 1_{av}}}{\Delta\sigma_{p\varepsilon 1}} = \frac{\left(\frac{2}{3}\right)^n \cdot k \cdot \varepsilon_1^n}{\frac{1}{2}\cdot k \cdot \varepsilon_1^n} = 2\left(\frac{2}{3}\right)^n \tag{31}$$

Importance of rolling zone invariable 2/3 is applied to the calculation of an average normal contact stress $\sigma_{n,av}$ by means of Korolev equation [18]

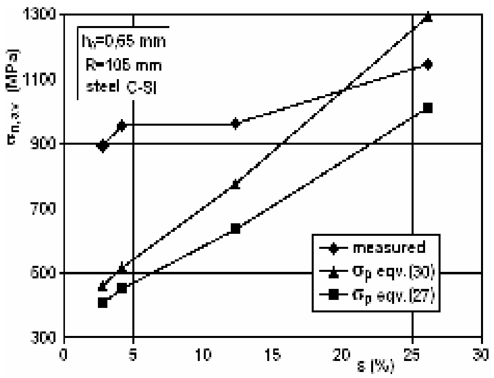


Fig.5 Dependence of average normal contact stress on deformation

$$\sigma_{n,av} = \sigma_p \cdot \frac{e^m - 1}{m} \tag{32}$$

where constant m is established by formula

$$m = f \cdot \frac{l_d}{h_{av}} \tag{33}$$

as h_{av} presents average thickness $h_{av}=0.5(h_0+h_1)$. Verification is realized on results from experiments given by Kvačkaj at al. [19]. Material of sheets used in experiments was steel C-Si with thickness of h_o=0.65 mm. Cold rolling of sheets (one pass) was proceeded by rollers with radius R=105 mm, and rollers circumferential speed was 0.66 m/s. Resistance deformation σ_p is calculated from strain hardening curve given by the equation (34) [19].

$$\sigma_p = 226 + 68.8 \cdot \sqrt{\varepsilon} \tag{34}$$

In order to get the first calculation of average normal contact stress from the equation (32), resistance deformation was established from equations (34) and (25). In order to get the second calculation, the value 2/3 was applied to rolling zone invariable; and resistance deformation was assessed from equation (28). Curves constructed from calculated values are seen in **Fig. 5**. The values obtained from experiments [19] are displayed in **Fig. 5** as well.

6 Conclusion

From the presented study of deformation in rolling zone the following results were found out:

- average value of relative deformation ε_{av} in rolling zone is not equal to total deformation for one pass;
- distribution or relative deformation ε_x along the length of rolling zone l_d governs elliptic distribution;
- ratio of average value to total value of relative deformation $\varepsilon_{av}/\varepsilon$ in rolling zone is possible to be considered to be invariable equal to 2/3;

- rolling zone invariable which equals to 2/3 is available for the computation of resistance deformation from strain hardening curve of cold rolling. Resistance deformation calculated in this way represents real value of resistance deformation more accurately.

Average value of relative deformation ε_{av} in rolling zone was established as integral value which more accurately describes quantity of deformation.

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